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AN

INTRODUCTION TO THE STUDY

OF

GENERAL PHYSICS

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BY

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PREFACE

This volume now forms Book I of my 'Introduction to the Study of Physics,' meant for the Intermediate Course of Indian Universities.

In the treatment of the subject herein I have been guided in the main by the Intermediate Syllabus in Physics of the different Indian Universities. I have taken care, however, not to omit to insert elementary descriptions of some important phenomena or informations of a general character, such as Capillarity, Flying Machines, Molecular Motions in Gases, Liquids and Solids, etc. ; articles dealing with these have been mostly marked with asterisks.

As for the size of the present treatise, which may appear to some to be rather big, I beg to note that in Part I which covers about half of the book, I have tried to give a systematic and clear exposition of the fundamental principles of Mechanics, which are so necessary in the study of Physics.

To make the book further useful, no pains have been spared in inserting a large number of experiments, printed in smaller types for ready distinction, numerous illustrations, and sets of

exercises, one at the end of each chapter, containing examples of a somewhat typical character as well as those set in the University Papers in the Intermediate Examination.

Advantage has been taken of a fresh edition of the book in adding some figures of general interest and in making slight alterations in the body of the book for the better elucidation of the subject treated.

I take this opportunity to acknowledge my indebtedness to the several elementary text-books on Physics available in the market.

My thanks are due to the Managing Proprietors of *Bani Press, Calcutta* for the interest they took in the printing of this book and for placing every facility in their press at my disposal.

CALCUTTA,
July, 15, 1925.

RAJANIKANTA DE

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CHAPTER I.

INTRODUCTION.

1. **Science.**—The word SCIENCE (from *Scio*, to know) originally meant knowledge. But the word has now come to mean a *system of exact and co-ordinated knowledge.*

Science and common knowledge.

No line, however, can be drawn between common knowledge of things and scientific knowledge. In strictness, all accurate knowledge is science. "Science and common-sense are," as Prof. Huxley observes "not opposed, as people sometimes fancy them to be, but science is perfected common sense."

Observation

The common knowledge is first obtained through the five senses of sight, smell, touch, taste and hearing. That knowledge must next be extended by careful *Observation*, *Experiment* and *Reasoning*. The methods of observation and experiment are again nothing new to us, for we are, all of us constantly making observations every day in our life and making experiments upon one thing or another. A scientific **Observation** differs from a common observation in being at once full, precise and 'free from unconscious inferences.' It should be made with the assistance

of exact measurements, wherever possible. A scientific **Experiment** is likewise a careful observation performed by watching what happens when the known conditions of an event of nature are *artificially* produced, or when some of the antecedents are altered. By an accurate **Reasoning** we learn how to state the results of our investigations in some general rules, called the *Laws of Nature*. A **Law of Nature** is thus a statement of the invariable order in which an event follows from certain antecedents. It is not a cause, but 'the expression of a definite connection between a cause and its effect.' For example, it is a law of nature that bodies, whenever unsupported, fall to the ground; the cause is the existence of an attraction between the earth and the things on its surface. A law of nature is thus the result of our observations; it states that from a certain set of circumstances a certain result always follows,—in other words, —*Nature repeats herself*.

Experiment.

Reasoning.

Laws of Nature.

The Physical Science (from Greek *Physike*, natural) means the knowledge of the Laws of Nature obtained by Observation, Experiment and Reasoning.

As a result of scientific knowledge men have come to discover also that nothing happens in nature by *Chance* or *Accident*. When we say that an event happens by a chance, we admit indirectly that we do not know all the antecedents of the event. "Chances and accidents are only *aliases* of ignorance."

Chance means ignorance.

2 Scope of Physics.—The Science of Physics, in the widest sense of the term, may be said to study the properties of Matter and Energy.

Matter is the material or stuff which all bodies are made of, and which are perceptible by us through two or more of our senses. It possesses certain fundamental properties, *e. g.*, extension, inertia, gravitation etc., which shall be considered later on (Chap. XII).

Energy is the capacity of doing work in bringing about any change in the state of matter against a resis-

tance, *i. e.*, against a force which tries to oppose any such change. The meaning of the term will be made clearer when the various forms of energy will be studied (art. 91).

According to the definition given above in arts. 1 and 2, the Physical Science aims at the study of the whole of Nature, *i. e.*, all the phenomena of the material world. As such, it may better be called **Natural Philosophy**, for it includes within its domain the various departments of *Mechanics*, *Astronomy*, *Zoology*, *Botany*, *Chemistry*, *Geology*, *Mineralogy*, etc., etc., each of which deals with separate classes of phenomena of the material world.

Departments of science for special studies.

It is now usual however, to restrict such a wide scope as has been mentioned above. Thus the phenomena of the growth of animals and plants depending on vital forces have been grouped apart under the domains of *Zoology* and *Botany* respectively; those involving the study of the nature and movements of the heavenly bodies constitute the science of *Astronomy*; the study of the minerals and that of the constitution of the earth's crust form the domains of *Mineralogy* and *Geology* respectively and so on.

Physics and Chemistry.

Physics, in the limited sense, discusses those properties only of matter which depend simply upon the state of bodies (see art. 99) and *not* on their constitution. The latter, viz., those properties of matter which are concerned with the composition of bodies and the interaction of one kind of matter with another kind are discussed in the science of **Chemistry**.

Expt. I. Heat a quantity of water in a flask. Lead the steam generated through a long pipe into a condensing flask. Note that the water formed from the steam is exactly the same thing as the water, from which the steam is formed.

The passage of water into steam and the return passage are physical phenomena.

Expt. 2. Rub a piece of steel with a magnet. The steel acquires the property of attracting iron filings to it.

Here the steel has simply acquired a new property ; nevertheless it remains steel.

Expt. 3. Pass an electric current through the thin metallic wires of an incandescent lamp. The wire is first heated up, and glows emitting light and heat. On stopping the current, the wire cools down and resumes all the properties which it had before.

In all these cases the substances do not undergo a permanent change : the change is , simply a *temporary* one. These are, therefore, instances of *Physical phenomena.*

Expt. 4. Burn a chip of wood. As the result of combustion a part of it passes into the atmosphere in the form of vapour and gas, while another part is left behind as residue in the form of ash and charcoal, which are different from wood.

Thus iron rust is not iron ; gun-powder after the flash is no more gun-powder. In these cases the substances which are charged, disappear altogether giving rise to the formation of *new* substances with entirely new properties. Such changes are *Chemical Phenomena.*

It will be seen later on that the line of demarcation between Physics and Chemistry has never been a clearly marked one.

3. Subdivisions of Physics—The science of Physics is usually divided into the following branches :—

(i) **General Physics**—dealing with the general laws of motion of bodies (MECHANICS) and the properties of MATTER.

(ii) **Acoustics**—studying the cause, the propagation and the nature of Sound, and the relation of tones in a music.

(iii) **Heat**—including RADIATION. Studies the effects of application of heat on bodies and the different ways of transmission of heat from one body to another.

(iv) **Optics**—studying the phenomena of LIGHT.

(v) **Magnetism and Electricity**—studying two very closely related subjects

The phenomena of Sound, Heat, Light, Magnetism and Electricity are those of the different forms of energy of which the Sound-energy is due to the wave-motion in a *material* medium, while the other forms are associated with the vibrations in a supposed medium called *Ether** by the scientists.

The Ether

The **Ether** is a very subtle and elastic medium which is supposed to fill all space and to permeate all matter. It is probably devoid of weight. It is quite unlike any gross form of matter with which we are acquainted. In some of its properties it nearly resembles an exceedingly rarefied gas ; in others, an elastic solid in the extremely attenuated form !

The relation between Matter and Ether is rather of an uncertain nature. The modern scientists think it probable, however, that "some of the properties of matter are determined by its relation with the ether with which it is associated."

4. Subdivisions of General Physics.—In the present treatise we make an elementary study of the portion of *General Physics* which again may be divided into two parts :—

PART I—**Mechanics**, or the Science of MOTION.

PART II—**Properties of Matter.**

Mechanics.

The term *Mechanics* was originally used by Newton to designate the *Scince of Machines and the Art of making them.* It is now, however, generally applied to mean the *Science of Motion and Equilibrium of bodies acted on by forces.*

Mechanics can generally be divided into the following two branches :—

* This is not to be confounded with the very tangible liquid called *Ether* by chemists.

(1) **Kinematics**—or THE SCIENCE OF MOTION studying the principle of motion itself,—apart from any consideration of the body moved.

(2) **Dynamics**—or THE SCIENCE OF FORCE, studying the properties of the forces as deduced from those of motion. The forces may either produce motion in a body (**Kinetics**), or may be in equilibrium and keep the body at rest (**Statics**).

PART II,—dealing with the Properties of Matter,—may be conveniently subdivided into

(1) **General Properties of Matter**, and

(2) **Special properties of Matter**, *viz.*,

(*a*) Properties of Solids

(*b*) „ „ Liquids

(*c*) „ „ Gases.

Exercise.— I

1. How does scientific knowledge differ from common knowledge ?

2. Explain what are meant by *observation* and *experiment*.

3. What is a *law of nature* ? Give two examples of such a law.

4. What is a *phenomenon* in nature ? How is it explained ? What does *Physics* aim to study ?

PART I.

MECHANICS.

CHAPTER II.

UNITS AND MEASUREMENTS.

5. Units—The study of every science involves measurement of some quantities. To measure a quantity we must fix on some definite quantity of the *same kind* as the standard of reference, which is called the **Unit** of that quantity. Then the whole quantity is to be expressed in terms of this unit.

The number of times the unit is contained in the whole quantity is called the **Measure** of that quantity.

Thus when we say that a road is 5 ft. long, we imply that a certain length, called the foot, has been taken as the unit, and that the number of such units in the length of the rod is 5. By 2 seers of rice we mean, that if the unit of mass be 1 seer, the measure of the mass in the specified quantity of rice is 2.

The choice of unit is, of course, arbitrary ; but to avoid confusion certain definite units of different quantities are generally agreed upon either by *usage* or by *law.*

It is also evident that the measure of a quantity depends on the unit chosen. Thus in 32 annas, the unit is an anna and the measure is 32. But if a rupee = 16 annas be taken as the unit, the measure will be 2. By a change of units is meant the same thing as the *Reduction* in Arithmetic.

6. Fundamental and Derived Units—For measuring the different kinds of quantities different units must be used. These may be selected in any arbitrary way. With the progress of the Physical Science, however, it has been realised that there are certain relations that exist between different kinds of physical quantities.

If an arbitrary unit were selected for the measurement of every physical quantity, without any consideration of the inter-relation of the quantities, great confusion would result from the complicated relations between the units selected.

Selection of necessary units.

For this reason it has been thought convenient to select units of certain quantities as the **Fundamental Units** and then to derive the units for all other quantities from these fundamental units by means of carefully framed definitions. The units derived in this way from the fundamental units are known as the **Derived Units.**

Fundamental and Derived units.

It is found that in order to build up such a system of units, the fundamental quantities need not be more than three in number. The three fundamental units generally chosen are—

The three fundamental units.

the unit of **Length** (art. 7)
the unit of **Mass** (art. 9) and
the unit of **Time** (art. 12).

All other units of the system are derived units and may be expressed in terms of these. Thus a *unit area* is defined as the area of a square, of which the side is of unit length, and is, therefore, derived from the unit of length : again a *unit velocity* is that of a body which moves over unit length in unit time, and hence is derived from the units of length and time, and so on.

There are two recognised systems according to which the units are chosen : one, the *French* or C. G. S. system, and the other, the English or F. P. S. system. In the **F. P. S.** system the foot the pound and the second are taken as the units of length, mass and time respectively. In the **C. G. S.** system which is in common use in France and is more conveniently adopted for all scientific purposes, the units of length, mass and time are taken to be a centimetre, a gramme and a second respectively.

The two systems of units.

7. Unit of Length—The most convenient unit of length in common use in England is the **foot** (*ft.*). A foot is one-third of a yard. The **Yard** is defined by an Act of Parliament as the straight line or the distance between two marks on a certain bronze bar, now kept at the Standard Office of the Board of Trade in London, at a temperature of 62° Fahrenheit.

The Yard.

The foot, the inch and its subdivisions are submultiples of the Yard, while the furlong, the mile etc., are the multiples.

The unit of length in the C. G. S. system is the **Metre**. It was originally defined by the French Republic (in 1801) as the ten-millionth ($1/10^7$) part of an arc of the earth measured from the North Pole to the Equator along the meridian passing through Paris. The metre is now defined. however, as the length of a certain platinum rod at 0°C, kept at the Archives in Paris. This is now known to be not exactly one ten-millionth of a quadrant ; it is too short by about 2 *mm.*

The Metre.

A metre is divided into tenths (*Deci*), hundredths (*Centi*) and thousandths (*Milli*) ; thus

1 metre (*m*) = 10 Deci-metres (*dm.*)
= 100 Centi-metres (*cm*)
= 1000 Milli-metres (*mm.*)

For the multiples of a metre we have

10 metres = 1 Deca-metre (*dm.*)
100 „ = 1 Hecto-metre (*Hm.*)
1000 „ = 1 Kilo-metre (*Km.*)

For laboratory measurements, the unit of length generally adopted is the *Centimeter.*

The relative magnitude between the units in the two systems is shown below with sufficient accuracy :—

1 metre = 3·37 inches = 1·09 yds.
1 inch = 2·54 cm.
1 mile = 1·6 kilometre.
1 kilometre = 0·62 miles, *i. e.* = 5/8 of a mile approx.

8. Measurements of Lengths.—For finding the length or distance between two points in physical measurements various appliances are used according to the accuracy desired. For all ordinary purposes a **Metre-Scale** can be used. It is usually made of box-wood or steel, and is generally graduated in inches and tenths of an inch along one edge or on one side, and in centimetres and millimetres along the other edge or the other side.

Expt. 5. Take a metre scale and examine it. Find from the scale the equivalent of a decimetre in inches.

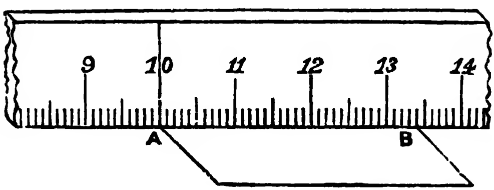

FIG. 1.
Use of a Scale.

Such a scale can be directly placed alongside the length to be measured (fig. 1.) and then the length is to be read off from the graduations of the scale. Where the direct application of the scale is not convenient, a pair of ordinary dividers or modified form of dividers, such as the *Beam-Compass* (fig. 2) or the *Simple*

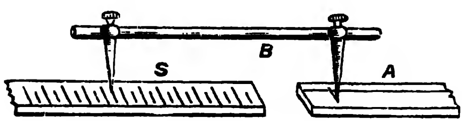

FIG. 2.
Beam-Compass.

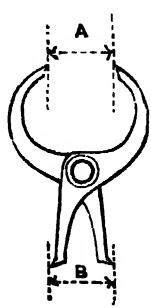

FIG. 3.

Calipers (fig. 3.) may first measure off the required distance and then referred to a scale. With a scale divided into tenths of an inch or millimetres it is possible with a little care and practice to measure approximately by eye a length within a hundredth of an inch or to a tenth of a millimetre.

In cases where a greater accuracy is wanted, the simple process of furthur subdivision of the scale does not help much. Other mechanical contrivances have to be adopted. Of these, the most commonly used are the *Vernier* and the *Micrometre Screw*.

The **Vernier** (fig. 4.) is a short, auxiliary scale that is used with the main measuring scale to estimate lengths to some particular fraction of the smallest division on the scale.*

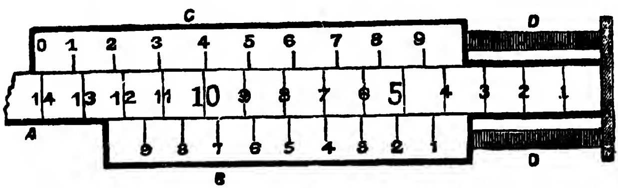

FIG. 4.
The Vernier.

It is so graduated that a certain length of the main scale corresponding to n equal divisions in the vernier is divided into either $n+1$ or $n-1$ equal divisions. Thus measurements can be made up to $1/n$ of the scale division, for

if s = value of one scale division
and v = ,, ,, ,, vernier ,,
then $(n \pm 1)s = nv$.
or $v = \frac{n \pm 1}{n} s$

$\therefore$ Diff. of s and $v = \frac{1}{n} . s$; this is called the ***Least Count*** of a vernier.

* See also *De's* Intermediate Practical Physics.

How to use a vernier.

The position of the pointer or the line marked zero on the vernier is read off from the main scale. To this is added the product of the least count and the number of divisions of the vernier where coincidence occurs with an opposite scale-division.

The principle of the vernier is applied to a number of measuring instruments, *e. g.*, the slide-calipers, the cathetometers, the travelling vernier-microscopes, the

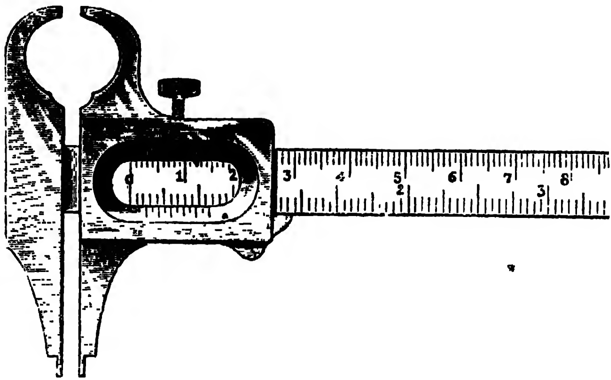

Fig. 5.—Slide-calipers.

telescopes, the spectrometers etc. etc. The simplest of these is the **Slide-calipers** (fig. 5).

A slide-calipers consists of a steel scale and two steel jaws One of the jaws is fixed at one end of the scale at right angles to it, while the other which is provided with a vernier and a fixing nut, is movable along the scale. When the two jaws are in contact, the zero of the vernier coincides with the zero of the scale. Hence when the jaws are taken apart, the gap between them is measured by the scale reading.

In the **Micrometre Screw** the motion of a screw is utilized.. This is simply a very accurately cut screw of a small pitch moving in a fixed nut. It is provided

with a large circular head, the circumference of which is divided into a convenient number of equal parts, say

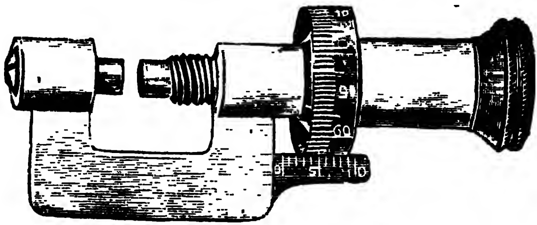

FIG. 6.—Screw-gauge.

50 or 100, so that a fraction of complete rotation can be read. To indicate the number of complete turns of the screw, a straight scale is fixed parallel with the axis of the screw. The theory of this is very simple. For one complete turn of the screw its point will move through a distance equal to the pitch of the screw, *i.e.*,—to the distance between the consecutive threads. Suppose the pitch of the screw is $\frac{1}{2}$ *mm*, and the head is divided into 500 equal parts. Hence the turning through one division on the head corresponds to a movement, forwards or backwards, of the screw-point through $\frac{1}{500}$ of $\frac{1}{2}$*mm* *i. e.*, ·001 *mm*. Hence readings accurate to a thousandth of a millimetre can be obtained with this micrometer screw. Two most commonly used instruments that work on this plan are the **Screw-gauge** (fig. 6.) and the **Spherometer** (fig. 7)*.

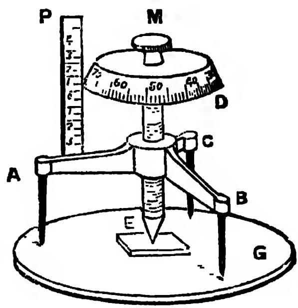

FIG. 7.
Spherometer.

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* For a full description of these two instruments see *De's* Intermediate Practical Physics.

9. Unit of Mass.—The mass of a body may be defined as the total quantity of matter contained in it. It does not depend on the volume or space which it occupies. Thus a piece of india-rubber when compressed has a different volume, but retains the same mass. The mass is altered only by changing the quantity of matter in the body,—in other words,—when the body gains or loses matter.

To measure any mass, we must select a unit of mass. The standard unit of mass in England is the **Pound** Avordupois (*lb*), which is the mass of a certain piece of platinum that is preserved, like the standard yard, at the office of the Board of Trade.

Multiples and Sub-multiples of the Pound.

1 ton	= 20 cwt.	= 2240 lb.
1 cwt.	= 4 quarters	= 122 lb.
1 stone	= 14 lbs.	
1 lbs.	= 15 ozs.	= 7000 grains.
	1 oz.	= 437·5 grains.

The unit of mass in the C. G. S. system is the **Gramme** or *gram* (*gm.*) which is one thousandth of a standard kilogramme. The standard kilogramme is defined to be the mass of a certain lump of platinum kept in the Archives at Paris. The gram was originally defined as the mass of a cubic centimetre of distilled water at 4°C. Accurate measurements have shown that the two units are not exactly the same; but for all ordinary purposes the latter statement is sufficiently nearly correct.

Multiples and Sub-multiples of the Gram.

1 kilogram (*kg.*) = 10 hecto-grams = 100 decagrams.
= 1000 grams.

1 gram (*gm.*) = 10 deci-grams = 100 centigrams
= 1000 milli-grams (*mg.*)

The relative magnitude of the pound and the gramme is given below :—

1 kilogramme = 2·204 pounds = $2\frac{1}{5}$ lbs. nearly
1 gramme = ·002 pounds = 15·4 grains.
1 pound = 453·59 grams.

10. **Mass and Weight**—It is necessary at this stage to understand clearly the distinction between mass and weight of a body. From our daily experience we are familiar with the fact that all bodies on the earth possess *weight*, due to which they constantly tend to fall to the earth. The real cause of the weight of a body is the attraction exerted between the earth and the body, due to which, the earth, having a much large mass, pulls the body towards it. The centre of the earth is the point from which its attraction may be regarded to be exerted.

The weight of a body is thus the result of the earth's attractive pull exerted on the body. Now the magnitude of this pull,—hence the weight of a body,—depends upon the mass of the body and upon its distance from the centre of the earth (art. 105). *At a given place*, therefore, the weight of a body is proportional to its mass ; in other words, bodies which are equal in weight are also equal in mass. On this basis masses are measured or *compared* by the process of **weighing**. Indeed, the *measure* of mass by weight is so general, that the two words are commonly used as synonymous.

Principle of weighing.

But the mass and the weight are not the same thing. While the mass of a body remains constant unless matter be added to or taken away from it, its weight may *vary from place to place* as its distance from the centre of the earth changes. For example, the weight of a body is found to increase slightly, when it is taken from the equatorial region to the polar regions.* Again a body is observed to lose its weight, though very slightly, when it

* For an explanation of this fact see art. 46.

is taken to a considerable height above the sea-level, as on a mountain top or in a balloon ascent.

11. Measurement of Mass.—It has been shown in the last article that as the weight of a body is directly proportional to its mass, any two masses are compared by the process of **weighing**. The instrument used for the purpose is the **Balance** (fig. 8).

The ordinary balance consists of a horizontal *Beam* (1 in fig. 8) balanced on a *Knife-edge* of a triangular prism fixed at its middle. The knife-edge which is made of steel or agate to minimise friction, rests upon a plate

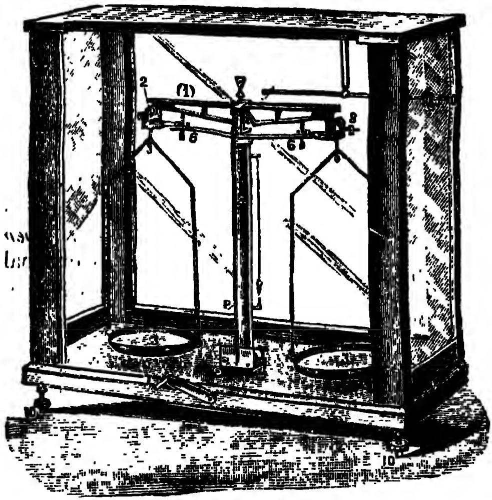

FIG. 8.
Balance.

of steel or agate fixed on the top of the supporting *Pillar*. At the extremeties of the beam there are two knife-edges turned upwards to support the inverted *Stirrups* (2,2 fig. 8) from which the *Scale-pans* are suspended. Hence

the *arms* of the beam measured from the fine knife-edge at the centre to those at either end are equal. A long *Pointer* P, which is attached at its upper part to the centre of the beam, oscillates with it ; when the beam is horizontal, the lower end of the pointer points to the zero mark of a graduated arc fixed on the pillar.

To preserve the sharpness of the knife-edges, the beam, when not in use, rests upon the *Arresting Arrangement*, the under surface of the pans just touching the base-board at the same time. To secure this, the central pillar which supports the beam can be lowered by means of a small *key* or an eccentric arrangement fixed at the base of the pillar and worked by a small handle shown at the front of the base-board. When the pillar is raised, the beam swings freely on the central knife-edge and the balance is ready for use.

The body to be weighed is placed on one pan and the standard weights, *i. e.*, the multiples and the submultiples of the unit mass are placed on the other pan until equilibrium is established, *i. e.*, the beam is quite horizontal*. From the principle of levers (art. 74), it follows that as the arms are equal, the two weights on the pans are equal. And as weight is proportional to the mass, we can say that the corresponding masses are also equal.

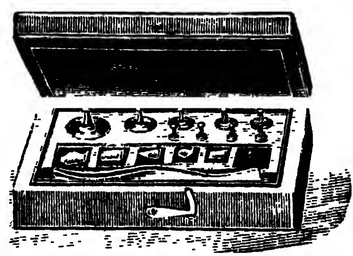

FIG. 9.

Weight-box.

The masses which are used as **standard weights** are contained in a weight-box (fig 9). The weights are

* For directions in using a balance see *De's* Intermediate Practical Physics.

Standard weights-

arranged within the box in an order, each being placed in a separate groove. The order of arrangement of the weights in an ordinary box is usually as follows—

(1) Brass weights,—

100, 50, 20, 20, 10 } *gms.*
5, 20, 1 }

(2) Fractional weights,—

	0 5,	2,	·2,	·1	*gms.*
marked	500,	200,	200,	100	*mgms.*
	05,	·02,	·02,	·01	*gms.*
marked	50,	20,	20,	10	*mgs.*

These fractional weights which are generally made of light aluminium, fit into separate compartments and are all covered with a thick glass slab. On one side, a pair of forceps is also provided.

Another kind of instrument which is sometimes used for a quick and approximate measurement of weights is the **Spring-Balance.** One form of it, represented in fig. 10 consists of a spiral spring, the upper end of which is secured to the top of the instrument, and the lower end is attached to a straight rod. The rod carries an *index* or a pointer which moves along a scale and has a hook attached to its bottom. When an object is hung upon the hook, the spring is stretched, until its elastic forces balance the weight of the body. The scale is previously graduated by hanging various known weights from the hook and marking their weights opposite the positions of the pointer along the scale The space between two such marks is subdivided into equal divisions, for the extension of the spring is found to be quite regular.

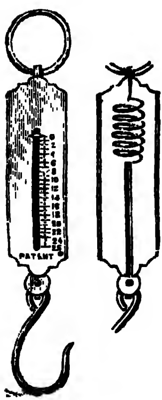

Fig. 10.
Spring-balance.

12. Unit of Time—The standard unit of time is derived from the rotation of the earth on its axis, which

is revealed to us by the apparent motion of the sun across the sky by day and of the stars by night.

The interval of time that elapses between two successive passages, called ***transits***, of the sun across the meridian of any place is called an **Apparent solar day**. By careful observations with accurate instruments it is found that the length of this apparent solar day is not always the same, but it slowly increases and then slowly decreases. After a year, *i. e*, $365\frac{1}{4}$ days the earth comes back to exactly the same position in her orbit with regard to the sun; and the length of the apparent solar day goes through the same cycle of variation from day to day. If however, we add together the lengths of all the apparent solar days and divide the sum total by $365\frac{1}{4}$, we get an average value of this interval which is called the **Mean solar day** or an ordinary day. The mean solar day is constant from year to year. It is this mean solar day whose subdivisions are measured by our clocks and watches, such that

$$1 \text{ mean solar day} = 24 \text{ hours} = 24 \times 60 \text{ minutes}$$
$$= 24 \times 60 \times 60 = 86400 \text{ seconds}$$

The **mean solar second**, which is $\frac{1}{86400}$ the part of a mean solar day, is the unit of time generally adopted in all physical measurements.

Astronomers use another unit of time called the **Sidereal second.** The interval of time that elapses between two successive transits of a fixed star across the meridian of a place is found to remain constant. It is called the *Sidereal day*. Astronomers use sidereal clocks indicating sidereal hours, minutes and seconds of a mean solar day,

The sidereal day is calculated to be equal to 23 hours 56 minutes 4·09 seconds of a mean solar day.

13. Measurement of Time—Perhaps the earliest method of reckoning the hours of the day was by means of the ***Sun-dial.*** A flat metal disc, with the hours of the day marked upon it in a certain order, has a large pin or style, as it is called, standing in the centre of the plate so as to cast a shadow on the right hour, so long as the sun is shining.

Other useful contrivances that come to be used to indicate shorter periods of time are the *Water-clocks* and the *Sand-glasses*, the construction of which are based on the rate of flow of water or fine sand through a small opening.

Towards the latter part of the sixteenth century (in 1584) GALILEO discovered the laws of oscillation of a pendulum (art. 86). He discovered that a pendulum which is simply a weight at the end of a string will always take the same time to swing backwards and forwards, so long as the string is of the same length. In 1658, HUYGHENS first applied the pendulum to regulate the motion of clocks.

The instrument generally used for the measurement of time is a **Clock** or a **Watch**. The mechanism in the clock is regulated in its motion by the oscillations of a pendulum, and that in a watch by the oscillations of a flat elastic spring, called the balance-spring. A **Chronometer** is an accurately constructed watch that keeps time with perfect regularity. Another variety of watch is the **Stop-watch** which has a large 'second' hand revolving over the main dial once in a minute and a smaller 'minute' hand revolving over a smaller dial once in 30 minutes. The watch is provided with a spring stud at the top by pressing which the 'second hand' is started, and again stopped at any time by pressing it a second time. On pressing the stud a third time, the second hand flies back to the zero position. An ordinary stop-watch generally measures time up to $\frac{1}{5}$th of a second. It is very convenient to use where a particular interval of time is to be noted.

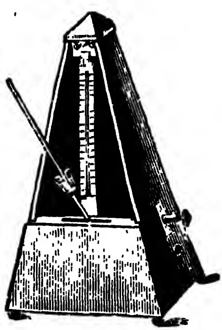

FIG. 11.
Metronome.

Another instrument, called the **Metronome** (fig., 11) is much used to mark the time in practising music.

It is virtually a pendulum provided with a ticking mechanism and is so constructed that its time of oscillation varies within certain limits. In fig. 11. the bob of the pendulum is just behind the wooden cover in front ; a small weight slides along the rod of the pendulum and can be fixed in any position. If the sliding weight is moved up, the pendulum swings slower ; and when it is moved down, the oscillations become quicker. Behind the sliding weight is a scale on which are written the numbers of ticks which the pendulum will make in a minute, when the sliding weight is at different heights along the rod. For example, if the weight be at the number 150, it means that 150 ticks would follow one another in 1 minute : hence the interval between two consecutive ticks is $\frac{60}{150}$ or ·40 of a second.

14. Measurement of Area—For all scientific purposes, the unit of area is a square of which each side is of unit length. In the C. G. S. system the unit of surface is a *square centimetre* (*sq. cm.*). The British unit of area in physical measurements is a *square foot.*

For multiples and submultiples, we have

1 sq. cm.	=	10 mm	×	10 mm	=	100 sq. mm
1 sq. metre	=	100 cm	×	100 cm	=	10,000 sq. cms
1 sq. yd	=	3 ft.	×	3 ft.	=	9 sq. ft.
1 sq. ft.	=	12 in.	×	12 in.	=	144 sq. in.

The numerical relation between a sq. cm. and a sq. in. is shown below :

1 sq. cm	=	0·155 sq. inch.
1 sq. in.	=	6·45 sq. cm

It is convenient to remember that 31 square inches are almost exactly equal to 200 square centimetres.

The areas of regular plane figures can be readily calculated by the application of geometrical formulæ on the measurements of necessary lengths. Thus

the area of

a Parallelogram	=	base × perpendicular ht.	=	ad.
a Triangle	=	$\frac{1}{2}$base × altitude.	=	$\frac{1}{2}bh$.
a Circle	=	$\pi\times(\text{radius})^2=\pi r^2$		where $\pi=3·1416$.

an Ellipse $= \pi \times$ semi-major axis $\times$ semi-minor axis.
the surface of a sphere $= 4\pi \times (\text{radius})^2$.
the curved surface of a cone $= \frac{1}{2}$ slant. height $\times$ circumference of base.

The area of an irregular plane figure may be experimentally determined by placing the figure on a sheet of cardboard or a thin metallic foil of uniform thickness, and then cutting it out and weighing. The weight of a known area of the same card or foil is then measured. From the comparison of the two weights the area of the figure is calculated.

Area by weighing.

Another method is to transfer the figure to a piece of squared paper and to count the number of small squares included within the figure. The product of this number and the known area of a small square determines the area of the figure.*

Area by squared paper.

15. **Measurement of Volume.**—The unit of volume for all physical measurements is the volume of a cube, each edge of which is of unit lengh. In the C. G. S. system the unit is the *cubic centimetre* (*c. c.*). For commercial purposes, the unit of volume is the **Litre,** which is 1000 *c. c.* Large volumes of a liquid or a gas are generally expressed in litres.

The British unit of volume is the *cubic foot.* The unit in which volumes of a liquid are measured is a *pint* such that

1 gallon = 4 quarts = 8 pints.
1 pint = 20 ounces.

The volume of any *regular* solid is obtained by the application of formulae and the measurement of the necessary dimensions of the solid. Thus the volume

Of a cylinder = area of base $\times$ height $= \pi r^2 h$.
Of a parallelopiped = height $\times$ area of base.
Of a pyramid $= \frac{1}{3}$ base $\times$ height.
Of a cone $= \frac{1}{3} \times$ area of base $\times$ height.
Of a sphere $= \frac{4}{3}\pi\ (\text{radius})^3$.

* See the Author's Intermediate Practical Physics.

The volume of a solid body, regular or irregular, may be easily obtained by the method of *displacement* of water. The solid is tied with a string and gently lowered into a tall jar, provided with a spout at the side and filled with water. The water displaced by it raises the level of water, and the surplus flowing off by the spout is collected in a graduated cylinder G (fig. 12), which measures the volume.*

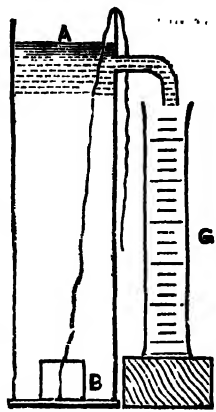

FIG. 12.
Vol. by displacement of water.

A more accurate method of finding the volume of a solid is by the application of *Archimedes' Principle* (art 141). A body is first weighed in air by a common balance and then in water. The loss of weight of the body in water measures the buoyancy in grammes which is numerically equal to the volume of the body in cubic centimetres.

The volume is also obtained by dividing the weight of a body by its known density from the relation

$$M = v\rho$$

where M is mass of the body, v its volume, and ρ its density (see art. 109).

The volume of a liquid is measured by the measuring vessels, *e.g.*, the measuring flask, pipette, burette, graduated cylinder etc. (figs. 13-16).† The *pipette* is mainly used for transferring small quantities of a liquid of known volumes. The *graduated cylinder* is used to measure rather a large quantity of a liquid to no higher accuracy than to the nearest whole cubic centimetre. The *burette* is used for measuring a given volume

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* See *De's* Practical Physics.

† See Prac. Physics.

of a liquid with greater accuracy; a glass *float* in generally placed inside a burette to help in taking the reading of the liquid meniscus.

FIG. 13.
Measuring Flask.

FIG. 14.
Pipette.

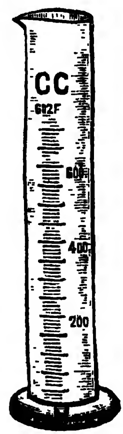


FIG. 15.
Graduated Cylinder.

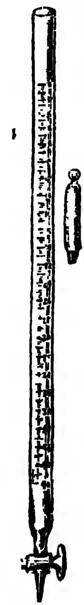


FIG. 16
Burette

16. Measurement of Angle.—The ordinary unit adopted for measuring angles is the **Degree**. A degree is $\frac{1}{90}$th part of a right angle. Hence 360° correspond to a complete rotation. A degree is divided into 60 minutes and each minute into 60 seconds.

The magnitude of an angle is found in practice by means of *Protractor* (fig. 17). A protractor is in its simplest from a divided circular scale.

Another unit of angle which is frequently used for theoretical purposes, is the **Radian**. It is the angle subtended at the centre of a circle by an arc of a circum-

ference taken equal in length to the radius. When the radian is used as the unit, an angle is said to be

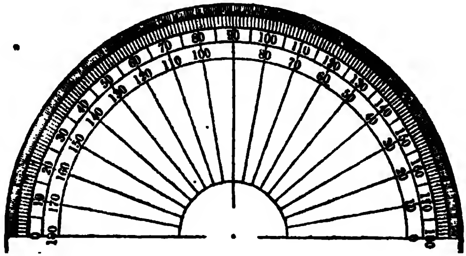


FIG. 17.
A Protractor.

measured in the *Circular Measure*. If θ is an angle subtended at the centre of a circle of radius r by an arc of length a, then $\theta = a/r$ radians.

Hence the angle subtended at the centre by the circumference of a circle of radius r is $2\pi r/r$ or 2π radians.

Hence 2π radians $= 360°$

Therefore 1 radian $= 360°/2\pi = 57°.2958$.

Some Useful Constants.

1	inch	=	2·54 centimetres.
1	metre	=	39·37 inches.
1	mile	=	5280 ft. = 80 chains.
1	fathom	=	6 ft.
69	miles	=	60 nautical miles.
1	knot	=	1 nautical mile per hour.
		=	6080 ft. per hour.
1	chain	=	66 ft.
1	sq. in	=	6·45 sq. cm.
1	cu. in.	=	16·39 cubic centimetres.
1	sq. metre	=	1550 sq. in.
1	cu. metre	=	61,025 cu. in = 35·31 cu. ft. = 1·308 cubic. yard.

Weight of 1 cu. ft. of pure water at 62°F = 62·3 lbs.
Do of 1 gallon " " = 10 lbs.
1 lb avordupois = 7000 grains = 453·6 grams.
1 kilogram = 1000 grams = 2·204 lbs.

Exercise.—II.

1. What are the three fundamental physical units and why are they so called ?

2. (*a*) How is the British unit of length defined ? Give its principal multiples and submultiples.

(*b*) What is a Metre ? Give the equivalents of the foot and inch in centimetres.

3. Explain the statement that we measure mass by weight.

4. Compare the British and French units of mass.

5. (*a*) What is a spring-balance and what is its advantage over an ordinary balance ?

(*b*) A set of observations taken with a spring-balance is given thus :

Wt. in the pan (in grammes),—

, 10, 20, 30, 40, 50, 60, 70, 90,

Extension in mm,—

6, 13, 20, 24 5, 30·5, 38·5, 42·2, 55.

By means of a graph find the approximate relation between these two quantities. Find the magnitude of the weight that will extend the above spring by 40 mm. [*C. U—1914.*

6. How will you find the volume of an irregular solid? How do you check it ? [*C. U.—1917.*

7. Explain the meaning of an ***apparent solar day*** and a ***mean solar day.***

8. Define mass, volume and density ; state the relation that exists between them.

What do you consider the best materials for the weights in an accurate box of weights ? Why is aluminium generally used for the fractions of a gram ? [*Pat. U.—1918.*

KINEMATICS

CHAPTER III.

MOTION.

17. **Kinematics.**—As has already been said (art. 4) that by kinematics is meant the study of motion as motion only,—without reference to the cause of the motion or the mass moved.

18. **Motion and Rest.**—A body is said to be in *motion* when it changes its position; thus a motion is *a change of position.* Conversely, if a body continues to occupy the same position for any length of time, it said to be at rest.

A little consideration will show us that the state of rest possessed by a body can only be apparent. The absolute rest, in other words, is unknown to us. A passenger in a railway carriage may be in a state of rest, relative to the train in which he is travelling, but really he is in a state of motion with respect to the trees, houses etc., past which the train rushes. These trees, houses and other objects again are not in absolute rest, for they are sharing the motion of the earth which is moving round the sun with considerable velocity and is also rotating about its own axis. The sun itself has not only a motion of rotation, but is moving through space carrying with it the whole of the solar system of which it forms the centre. Even the stars are observed to have a motion, which excludes the idea of the absolute rest.

Again, the absolute motion is equally unknown to us. To measure an absolute motion a point absolutely fixed

in the space is first of all necessary, which, however is impossible to be realised by us.

19. Motion of a Particle.—The motion of a body is easily understood if the motion of a material particle is first studied. By a *material particle* is meant that the particle of matter is infinitely small in volume, reduced practically to a point. The position of a particle may, therefore, be indicated by a point.

When a particle is in motion, the line joining its successive positions is called the *path of motion.*

The *displacement* of a moving point during a time is the straight line joining its initial position to its final position. The displacement posseses direction as well as magnitude.

20. Motion of a Body.—A body is supposed in Mechanics to be *a rigid* one, so that the size and shape of the body remain unchanged. Although a perfectly rigid body is not be realised in practice, the consideration of the dynamics of a rigid body is useful as an introduction to the study of the actual complex cases.

Motion of a body may be, in general, divided into two kinds. A body is said to have a motion of **translation**, when it moves in such a way that the motion of all the particles of which we may consider the body to be built up, is exactly the same. Hence any line within the body in its displaced state remains parallel to its original position. A train moving in a straight track,* a boat sailing in a straight line, the fall of a stone in a well are examples of translatory motion.

On the other hand, when a body moves about a fixed point or axis, round which the particles within the body describe concentric circles, it is said to have a motion of **rotation**. Thus the motion of a grind-stone on its axle, of a door on its hinges, of a pendulum about its point of suspension are instances of such motion.

* Neglecting the curvature of the earth's surface.

But the motion of a body is frequently a complex one which, however, can in all cases be considered as a combination of a rotation with translation. Instances of such motion are those of a wheel of a carriage, of a ball rolling along the ground, of a planet rount the sun etc.

When a body is undergoing a simple translation, the displacement is measured by the displacement of any particle in the body. In the case of simple rotation, the displacement of a body is measured by the *angular* displacement of a particle in it.

21. Velocity, Speed—.The velocity is the *rate of change of displacement.* Since a displacement includes not only a definite magnitude but also a definite direction in which the motion takes place, the measurement of a velocity of a particle involves the determination of

(1) the *speed* or the rate of motion which is measured by the amount of change of place in a given time and

(2) the *direction* of motion during this time.

Hence velocity means speed in some definite direction. Thus a body moving in a curved line may have a constant speed but never a constant velocity, for its direction of motion is continually changing.

22. Uniform Velocity.—The speed of a body may be *uniform* or *variable.* When a particle passes over equal distances in equal times, the velocity of the particle is said to be uniform.

When the velocity is uniform, it is measured by the distance transversed over in a unit of time If a particle passes over a space s in time t, the velocity v is given by the equation

$$v=\frac{s}{t} \quad \ldots \quad (1)$$

The unit of velocity is 1 cm. per sec. in the C. G. S. system and 1 ft. per sec in the F. P. S. system.

From (1), we get

$$s = vt \quad \ldots \quad (2)$$

Or distance traversed = velocity × time.

23. **Variable Speed**.—If a particle moves over unequal spaces in successive equal intervals of time, its speed is said to be *variable*. The variable speed at any instant is measured by the rate per second at which the distance is being traversed through a *small* interval of time containing that instant It is assumed that during this short interval the space does not appreciably alter, and then the formula $v = s/t$ applies.

Thus when we say that the speed of a railway train at a certain instant is 15 miles per hour, we mean that had the train moved with a constant speed equal to that it actually possessed at the time under consideration, for a very short time, say half a second, it would have moved through 22 ft *

24. **Acceleration**—When the velocity of a body changes, the rate of change of velocity is called the *acceleration.*

The unit of acceleration is the acceleration which in one second increases the velocity by 1 ft per second (in the F. P. S system), or 1 cm. per sec. (in the C. G. S. system), and is written as 1 ft. per sec², or 1 cm. per sec². If a body is moving at the rate of 10 ft. per sec. at any instant, and its velocity in one second later is 15 ft. per sec., the increase of velocity in one second is 5 ft. per second. The acceleration, therefore, is 5 ft. per sec. per sec.

N. B.—The words *per second* must be repeated as the unit of time is involved twice in the measurement of acceleration.

The word acceleration implies, in the strict sense of the word, an alteration of velocity in magnitude, or in direction, or in both magnitude and direction. It is however common to use the term acceleration with reference

* 30 miles per hour = 44 ft. per second.

to the changes in speed only, so that it means, in this sense, the rate of increase or decrease of velocity. If the speed be increasing, *e.g.*, when an express train starts from a station, or a stone is let fall from a height, the acceleration is said to be *positive*; while, if the speed be decreasing, *e.g.*, when a railway train going at full speed approaches a station, or a stone is thrown upwards with a velocity, the acceleration is said to be *negative*. Negative acceleration is, in the ordinary language, called **Retardation.** If a body be moving with a velocity of 60, 50 and 40 cms. per second respectively at the beginning of three successive seconds, the body would be said to be moving with a negative acceleration of 10 cms. per sec. per sec.

Acceleration may again be *uniform* or *variable*. In uniform acceleration, equal changes of speed occur in equal times. In other cases, it is variable. We shall consider here the uniform acceleration only.

Uniform Acceleration is measured by the change in speed that takes place in a given time, divided by that time, *i.e.*, by the change of velocity in an unit of time. Hence if the speed of a particle changes from u to u' during a short time t, the average acceleration f is given by the equation

$$f = \frac{u' - u}{t}$$

If t is small enough, the value of $\frac{u' - u}{t}$ may be taken as the acceleration of the particle at the particular instant considered.

25. Equations of Motion—The fundamental formulæ for the solution of problems on rectilinear motion with uniform acceleration can be put thus :—

Let

u = initial velocity of a particle moving in a straight line.

f = constant acceleration in the direction of motion.

v = final velocity at the end of time t.
s = whole space described during the time t.

Then, we have

for the final velocity v,

$$v = u + ft \qquad \ldots \qquad (1)$$

for the average velocity V,

$$V = \tfrac{1}{2}(u + v) \qquad \ldots \qquad (2)$$

for the distance traversed s,

$$s = ut + \tfrac{1}{2} ft^2 \qquad \ldots \qquad (3)$$

and finally from (1) and (3)

$$v^2 = u^2 + 2fs. \qquad \ldots \qquad (4)$$

Proof of (1).

Change of vel. per unit time = acceleration = f

∴ " " in t units of time = ft.

But since the particle starts with an initial velocity u, its final velocity $v = u + ft$.

Proof of (2) *and* (3):

Divide the time t into n equal parts, where n is a large even number. By taking n very large, the intervals will be so very small, that the velocity at the beginning or the end of an interval may be taken to be the velocity throughout the interval.

Now the velocity at the pth interval from the beginning

$$= u + fp.$$

And the velocity at the pth interval from the end

$$= v - fp$$

Hence the average velocity V for these intervals

$$V = \frac{u+v}{2}$$

It is evident that the average velocity has the same value for any other pair of such intervals.

Therefore the space actually traversed in time t is the same as

if the particle moved for the same time t with an average velocit V, *i. e.*,

$$S = Vt = \frac{u+v}{2}t = \frac{u+(u+ft)}{2}t = ut + \tfrac{1}{2}ft^2,$$

Proof of (4) :—

From (1) $\quad v^2 = (u+ft)^2 = u^2 + 2uft + f^2t^2,$

$\qquad\qquad\qquad = u^2 + 2f(ut + \tfrac{1}{2}ft^2)$

Hence by (3) $\quad v^2 = u^2 + 2fs.$

If the direction of an acceleration instead of being the same as that of the initial velocity, is exactly opposite to it, the general equations become

$$v = u - ft, \quad s = ut - \tfrac{1}{2}ft^2 \text{ and } \quad v^2 = u^2 - 2fs.$$

If the moving particle starts from rest, we have $u = 0$, the general formulæ take the simple forms

$$v = ft, \qquad s = \tfrac{1}{2}ft^2 \text{ and } \quad v^2 = 2fs.$$

26. Graphical Representation of Motion— The change in the speed of a body can be represented by a graph, called the **Speed Curve.** Let OX (fig. 18) be a line taken to represent time. Let a number of equal

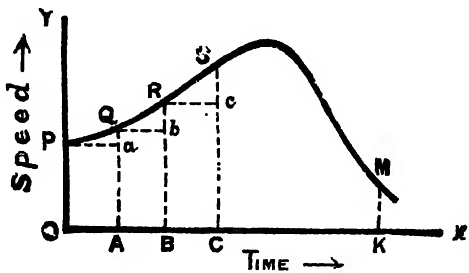


FIG. 18.

Speed Curve.

parts OA, AB, BC, etc, represent equal intervals of time, say one second. Let the speed be represented by the line OY. Suppose we commence to count the time from O

and let the speed with which the body is moving at this instant be represented by the length OP on the speed line. Similarly, AQ represents the speed at the time OA, BR the speed at OB and so on. Similarly, if all the corresponding verticals be drawn for the moments of time intermediate between O, A, B etc. and their extremeties be joined, the line PQRS...M is called the *Speed Curve*.

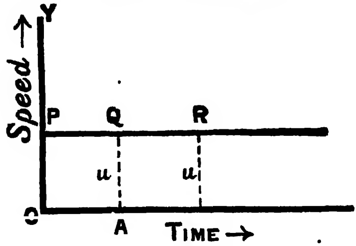


FIG. 19.

It must not be supposed, however, that the curve of speed is the same thing as the path of a body. Motion might take place in a straight line, and yet the speed curve might be represented as in fig. 18. The graph is merely a representation of the increase and decrease in the rate of motion at successive intervals of time.

Speed Curve and Path of a body.

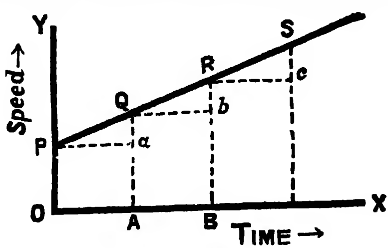


FIG. 20.

Uniform Motion—In a uniform motion the speed at different periods of time remain the same. The speed curve becomes a straight line parallel to the time-axis (fig. 19).

Uniformly Acceleration Motion—In fig. 20, the ordinates AQ, BR etc., represent the velocities at equal intervals of time, say a second. If we draw the lines P*a*, Q*b* parallel to the X-axis, Q*a*, R*b* will represent the increase of velocity, *i. e.*, the acceleration. When these are equal, the graph is evidently a straight line.

27. Representation of Velocities—The velocity of a moving particle can be conveniently ***represented*** by a straight line. For this purpose, a straight line may be taken in the direction of the velocity, and of such a length that it contains as many units of length as there are units of speed in the velocity considered. In order to show the sense of the direction, an arrow-head may be drawn on or by the side of the line.

28. Composition of Velocities—We have now to consider the case where a body may have at the same time more than one velocity. For example, a man moving about in a train has not only the velocity of the train, but an additional velocity of his own. Again, a person walking on the deck of a ship has an actual velocity which may be said to be compounded of three velocities *viz.*, the rate and direction of the ship's motion, those of the current and those of his own motion on the

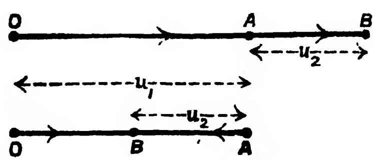


FIG. 21.

Velocities in the same line.

deck. (When a body has two or more simultaneous velocities, the velocity with which it actually moves is called the **Resultant** velocity, and those several velocities are called **Components.**) The process of finding the resultant velocity when the component velocities are given, is called *compounding* the velocities.

To compound velocities in the same straight line – If a body moves with several uniform velocities in the same straight line, the resultant velocity will be the algebraical sum of the component velocities. Thus, if a body tends to move from O to A in one second due to a velocity u_1, and from A to B in the same line in one second due to a second velocity u_2, then at the end of one second the body will be found at B, as if it had moved with a velocity $u_1 \pm u_2$. Suppose the velocity of a stream is 2 miles per hour and a man can row a boat through still water at 8 miles an hour, then the actual velocity of the vessel is 6 or 10 miles an hour according as the vessel is sailing up or down the stream.

In the case of variable velocities, *i. e.*, velocities with acceleration in the same line, the above remark is also true, for an acceleration is nothing but the additional velocity gained per second.

To compound velocities not in the same straight line.— If a body have two different velocities in different directions at the same time, it is evident that the actual motion of the body is along neither of these directions, but is along a line intermediate between them. Thus if a man rows a boat at right angles to the current of a river, the actual course of the boat is a line crossing the river in an oblique direction from one bank to the other.

Let the two simultaneous velocities be represented by the lines OA and OB (fig. 22), and let their magnitudes be u and v. Now we may imagine the body moving with the velocity u along OA, while the line OA moves with velocity v parallel to itself. At the end of one second, the particle would be at A due to the first motion, but owing to the motion of the line OA, the point A at the end of one second will have come to C. Thus the body will be at C.

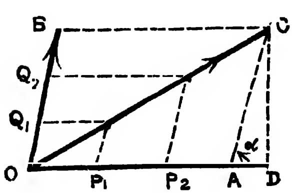


FIG. 22.
Parallelogram of Velocities.

Now, since the component velocities are uniform *i. e.*, constant in magnitude and direction, their resultant must be a uniform velocity. In other words, the velocity of the body from O to C must also be constant in magnitude and direction. Hence the straight line OC represents the resultant velocity.

We have, from the above, a rule for finding the resultant of two velocities, known as the **Parallelogram Law.** *If a particle possesses simultaneously two velocities represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, they are equivalent to single resultant velocity, also represented in magnitude and direction, by the diagonal of the parallelogram passing through that point.*

Expression for the Resultant Velocity.—Let OA and OB represent the velocities u and v in directions which are inclined to each other at an angle α. Therefore, the angle BOA in fig. 22. is α. Then we have, by trigonometry,

$$OC^2 = OA^2 + AC^2 - 2OA.AC. \cos OAC$$

Hence if OC represent the resultant velocity w, we have

$$w^2 = u^2 + v^2 + 2uv \cos \alpha$$

since $OAC = \pi - \alpha$ and $\cos(\pi - \alpha) = -\cos \alpha$. If CD is drawn perpendicular to OA, then

$$\tan COA = \frac{CD}{OD} = \frac{CA \sin CAD}{CA + CA \cos CAD}$$

$$= \frac{v \sin \alpha}{u + v \cos \alpha}$$

Hence the resultant velocity w which is given by $\sqrt{u^2 + v^2 + 2uv \cos \alpha}$ is inclined at an angle $\tan^{-1} \dfrac{v \sin \alpha}{u + v \cos \alpha}$ to the direction of the velocity u.

If a body possesses simultaneously more than two velocities u, v, w etc., the resultant velocity may be

obtained by the repeated application of the Parallelogram Law. Thus in fig. 23, the resultant of the velocities u and v represented by OA and OB respectively is R_1, acting along OM. Again the resultant of this R_1 and a third velocity w represented by OC is R_2

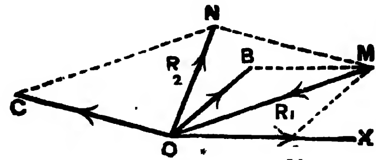


FIG. 23.
Resultant of several velocities.

along ON, which thus represents the resultant of the three velocities. And so on.

In other words, from any point O draw OA to represent the velocity u; from A draw AM parallel and equal to OB, representing the velocity v and from M draw MN parallel and equal to OC, representing the velocity w, then ON represents the resultant of the three velocities.

Similarly, any number of velocities may be compounded.

The Parallelogram Law applies in the case of accelerations too. If a body have two different uniform accelerations along two straight lines, the distances OP_1, P_1P_2 etc. (fig. 22) traversed over in successive equal intervals along OA will not be equal, but will be proportional to those along OB, *viz.*, OQ_1, Q_1Q_2 etc. Hence OC will represent the resultant accleration (fig. 22.).

29. Composition of a Uniform Motion with a Uniformly Accelerated Motion.—Suppose a body starting from rest at O moves with a uniform velocity u along OX and with a uniform acceleration

f along OY. Let us find the path actually traversed by the body.

Due to the uniform velocity u along OX the position of the body at the end of successive seconds will be P_2, P_3 etc, where $OP_1 = P_1P_2 = P_2P_3$ etc.

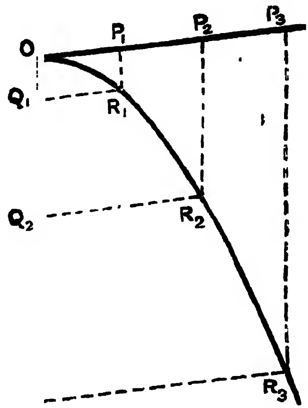


FIG. 24.

If the particle, starting from rest, were moving with the acceleration f along OY alone, the spaces traversed in 1,2,3, etc., seconds, would be obtained by making t successively 1, 2, 3, etc, in the equation $s = \frac{1}{2}ft^2$ (art. 25). Hence at the end of the *1st*, *2nd*, *3rd* etc., seconds the particle would be at distances $\frac{1}{2}f$, $\frac{1}{2}f.\ 2^2$, $\frac{1}{2}f.\ 3^2$, etc., from O measured along OY. Let Q_1, Q_2, Q_3, etc. be taken on OY such that $OQ_2 = 4 \times OQ_1$, $OQ_3 = 9 \times OQ_1$ and so on. So after three seconds the body will be at R_3. But its actual path is not along the diagonal OR_3; it is along a curved path $OR_1R_2R_3$....

An instance of such a path is the path of motion of a projectile thrown horizontally with an initial velocity. The path is the resultant of the composition of the horizontal uniform velocity and the uniform acceleration in a vertical direction due to the attractive pull of the earth.

Expt. 6. Note the path of motion of a water-jet coming out of a pipe connected with a street hydrant.

Take a syringe and draw water up. Holding the syringe horizontally, force the piston inwards. The water jet describes a parabola in air before it comes to the ground.

30. Resolution of a Velocity—It is sometimes convenient to replace the actual velocity of a body by two or more other velocities. In a such case, the velocity is said to be *resolved* into component velocities. Let a velocity U be represented by a straight line OC (fig. 25).

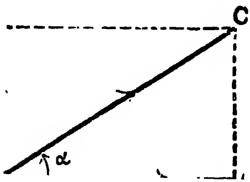


FIG. 25.

It is required to find its *resolved parts*, one along a direction OX and the other along any other direction OY. Through C draw CA parallel to OY, and CB parallel to OX; then by the parallelogram of velocities, the two velocities u and v, represented by OA and OB respectively, would have a resultant OC, so that we may replace the velocity U by the two velocities u and v. Any given velocity may be resolved in an infinite number of ways.

The most important case in practice, however, is that in which the two components are at right angles. Let the component u make an angle α with the resultant velocity U (Fig. 25).

Then $$u = U \cos \alpha$$
$$v = U \sin \alpha$$

31. Resolution of an Acceleration—In fig. 26, if OC represents any acceleration, OA and OB will equally represent the resolved parts of this along the two directions OX and OY.

Consider a particle moving in a path ABCD. The direction of motion of the particle at any instant is along the tangent to its path at the point. Let V_1 and V_2 be the velocities of the particle at the points B and C (Fig. 26).

Let OP and OQ represent in magnitude and direction these velocities V_1 and V_2.

The line PQ then represents the change of velocity during the interval in which the particle comes to C from B. When this interval is taken very small, *i. e.*, C is very near to B, PQ will represent the acceleration of the particle at B.

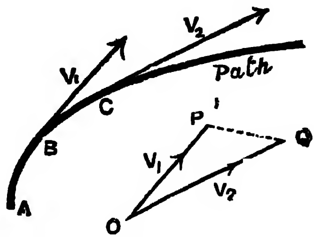


FIG. 26.

This acceleration may be resolved into two components at right angles to each other, the one in the direction of the tangent to the path and the other at right angles to the tangent. The former component is called the **Tangential Acceleration,** which alters the *magnitude* of the velocity but not its direction, while the latter component, called the **Normal Acceleration,** alters the *direction* of the velocity but not its magnitude.

When a particle is moving uniformly in a circle, there is no tangential acceleration, for the magnitude of a velocity is constant. The only acceleration which it possesses, is always at right angles to the direction of motion, *i. e.*, directed towards the centre of the circle. This normal acceleration causes the direction of the velocity of the particle to change continuously from point to point on the circle.

32. Relative Velocity—It is sometimes desirable to determine the motion of a body relative to a second body which is itself in motion.

The relative velocity of one particle A with respect to a second particle B is the rate *at which A changes its position relative to B.*

If the two particles are in motion with the same velocity, there is no change in their relative positions. Hence the velocity of one, relative to the other, is zero.

Thus a person in a train would, if he kept his attention fixed on a passenger in another train, moving in the same direction with the same velocity, and if he were unconscious of his own motion, consider the second person to be at rest.

When the particles are in motion with different velocities, the relative velocity of A with respect to B is obtained by compounding with the velocity of A, a velocity which is equal and opposite to that of B.

Let the velocity of the particle A in fig 27, be represented in magnitude and direction by AP and that of B by BQ. Now it is clear that the relative motion of two particles is not altered, if we impress the same velocity on both the particles: for example, the relative motion of two flies crawling on the window of a railway train is the same, whether the carriage be at rest or in motion.

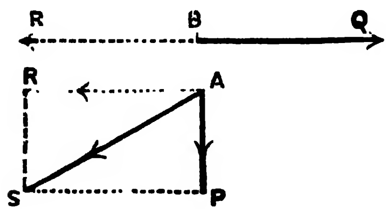


FIG. 27.

Imagine, therefore, that a velocity equal and opposite to that of B is impressed on the motions of A and B, represented by AR for the particle A and BR for the particle B. The relative motion is unaltered. The particle B is brought to rest, while A moves with a velocity which is the resultant of its own velocity and the reversed velocity of B. The apparent velocity of A is represented in the fig. 27 by AS.

When a man is running in a horizontal direction amidst a shower of rain, the drops which are falling say vertically, appear to him to strike his face in a slanting direction. The explanation of this is clear, if we apply to each a horizontal velocity equal and opposite to that of the man. This brings the man to rest. The

resultant velocity of a rain-drop is its apparent or relative velocity in magnitude and direction with respect to the man.

33. Angular Velocity—When a rigid body rotates round any straight line as its *axis of rotation*, all the particles, of which it may be regarded as built up, move in circles round points on the axis as centres.

The angular velocity of the rotation is measured by the angle which the radius, joining any particle to the centre of its path of motion, describes per unit of time. This angle is evidently the same for every particle in the body.

Let ω be the uniform angular velocity with which a particle describes an angle θ in an interval of time t seconds.

Then $\omega = \theta/t$ or $\theta = \omega t$.

The angle θ is always expressed in circular measure. If s be the arc described in time t by a particle situated at a distance r from the axis, with a uniform speed v, then

$$s = vt \quad \text{and} \quad \theta = s/r$$

whence $\theta = vt/r$

$\therefore \quad \omega . t/ = vt/r$

Hence $v = r.\omega$

The linear velocity of any particle in the body is thus directly proportional to its distance from the axis of rotation, and is found by multiplying the angular velocity by this distance. Thus in the case of a rotating wheel, a point on the rim has a greater speed than one on a spoke, for the former being situated at a greater distance from the axle has to describe a larger circle in exactly the same time by which the latter finishes its revolution.

The angular velocity may be expressed in terms of the number of terms per second. Let the body make n turns per second. Since in one complete turn the

angle turned through is 2π, the angle turned through in one second or the angular velocity $= 2\pi n$.

EXAMPLES :—

1. A body, dropped from a height, falls with an uniform acceleration. In the 4th and 5th seconds from the commencement of its fall it moves through 120 ft. and 184 ft. respectively. Find its initial velocity and the acceleration with which it moves.

Let u be the initial velocity, and f the acceleration.

Then the distance traversed in the 4th second

= distance fallen through in 4 secs. - distance fallen through in 3 secs.

$$= \left[u.4 + \tfrac{1}{2}f.4^2\right] - \left[u.3 + \tfrac{1}{2}f.3^2\right]$$

$$= u + \tfrac{7}{2}f = 120\ ft. \qquad \ldots \qquad (1)$$

Similarly, the distance fallen through in the 6th sec.,

$$= \left[u.6 + \tfrac{1}{2}f.6^2\right] - \left[u.5 + \tfrac{1}{2}f.5^2\right]$$

$$= u + \tfrac{11}{2}f = 184\ ft. \qquad \ldots \qquad (2)$$

From (2) and (1)

$2f = 64\ ft.$ whence $f = 32\ ft.$ per sec.

And $u = 8\ ft.$ per sec.

Exercise.—III.

1. Define Motion. Name and explain the different forms of motion.

2. Distinguish between Speed and Velocity.

3. Define Uniform Velocity and explain how it is measured.

4. What is meant by the statement that the acceleration of a particle is 32 ft. per sec. per sec. ?

5. A river, 1 mile broad, is running downwards at the rate of 4 miles an hour, and a steamer, moving at the rate of 4 miles an hour, wishes to go straight across. How long will the steamer take to perform the journey, and in what direction must she be steered? —*Lond Matric.*

6 A train, moving with a velocity of 30 miles an hour, stops in 5 minutes as it approaches a station. Express the retardation assuming it to be uniform, and taking a foot and a second as the units of length and time?

7. A ball is thrown out of a window of a railway carriage in motion. In what direction will it seem to fall to a passenger in the train and also to a person on the ground?

8. At the earth's equator the hot air ascends, and is replaced by cold air which blows in along the ground from the poles. That which comes from our hemisphere blows from the north-east instead of from the north. Explain this. —*Lond Matric.*

9. A ship is sailing due north at the rate of 4 ft. per sec.; a current is carrying it due east at the vertical mast at the rate of 2 ft. per sec. What is the velocity of the ship, and what is the velocity of the sailor, relative to the sea-bottom?

10. A man is walking in the north-easterly direction with a velocity of 6 miles per hour. Find the components of his velocity in directions due north and due east respectively.

11. Show how to find by a graphical construction drawn to scale and also by calculation the resultant of the following velocities communicated to a point, viz, 2 ft. per second in an easterly, 20 ft. per second in a north-easterly, and 30 ft. per second in a northerly direction respectively.

DYNAMICS

CHAPTER IV.

FORCE.

34. Dynamics.—We have now to consider motion in connection with the quantity of matter moved and with the cause of producing it. In other words, we shall have regard to the *mass* of the moving body, and to the *force* that causes or modifies its motion.

35. Force.—A material body is said to be inert, *i. e.*, it cannot of itself change its state of motion or of rest. Whenever there is such a change, it must be due to some cause from outside. Force is the name given to this unknown cause, and may be defined as *whatever causes, or tends to cause, motion or change of motion in a body.*

Force is best defined in the first Law of NEWTON'S three Laws of Motion which are accepted as the foundation on which the relations between matter, mass and force are established.

Laws of Motion.

These laws do not admit of direct experimental proof. Their truth, however is admitted from conviction drawn from general observations. In other words, assuming the laws to be true, we can solve the various complicated problems of Mechanics. The accordance of observation with the solution is more exact, the more completely we apply the laws. They are, in fact, the generalisation of some fundamental principles which Newton recognised in certain simple cases of motion.

36. The First Law of Motion.—Newton's first law of motion states that *a body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state.*

The law consists of two distinct parts. The first part of the law states that any body or piece of matter is unable by its own action to change its own state of rest or motion. If it be at rest, it must continue to be at rest ; and if it be in motion *i. e.*, if it be moving in a certain direction at a certain rate, it cannot, of itself, change its direction or the rate of motion. This property of matter is a general one and is called **Inertia**. (See art. 106).

The second part of the law serves to define *force*. It states that the state of rest or motion of a body will change only under a certain condition *viz.*, when a force is impressed upon it. We may, therefore, define force as that which tends to produce a change of motion in a body on which it acts (rest being regarded as motion with zero velocity), the change of motion including any change of direction as well as any change of speed.

Familiar examples of force are to be found in the pull of a horse on a cart, of a locomotive on a train, of a weight on a rope which supports it, in the push or thrust of the muscles on a bicycle pump etc., etc.

It is to be remembered, in the interpretation of the law that when a body is not acted upon by a force, it must be in one of the two states of (1) being at rest, or (2) having a uniform motion in a straight line, either of which signifies that there is *no change* in the motion of the body. Further, the forces which bring about a change in the state of rest or motion of a body are *impressed* or external forces, and not *stresses* or internal forces between the parts of the body, for these latter cannot change the state of a body taken as a whole.

The law can not, as has been said already, be verified directly, for it is impossible to remove the several external influences which always act on a body to impede its motion to a greater or less extent, but our experience warrants that the more completely a body is freed from the action of these influences, the less marked is the change in its state of motion. Thus, the more smooth the ground along which a marble ball is rolled, the further will it go.

37. The Second Law of Motion—*Change of motion is proportional to the impressed force and takes place along the straight line in which that force is impressed.*

Evidently, the term 'motion' is used in this law to mean a *measurable* quantity of motion or what is called the *Momentum*, for the law asserts that it is proportional to the impressed force. The **Momentum** of a body is a quantity measured by the product of its mass and its velocity. Thus

If m = mass of body

and v = its velocity at any instant.

then mv = **momentum** of the body at that instant.

It is a vector quantity and its direction at any instant is the same as that of the velocity at that instant. The unit of momentum is the momentum of a unit mass moving with unit velocity.

It is also easy to see that the element of time is implied in the law, for a certain force acting for two seconds, will evidently produce a change of motion twice as great as it can produce when it acts for one second. Hence, we may interpret the change of motion here to mean the change of momentum per unit time or the rate of change of momentum.

Secondly, the law affords a method of measuring force. Let u be the velocity of a body of mass m moving along a straight line under the action of a

constant force P, and v the velocity at any instant t units later on.

Change of momentum in time $t = mv - mu$

$\therefore$ Rate of change of momentum $= \dfrac{mv - mu}{t}$

By the law,

P is proportional to $m \times \dfrac{v-u}{t}$;

i.e. „ „ „ „ $m \times$ acceleration. ... (1)

Since P is constant in magnitude and direction, the accceleration is also constant.

Hence, *equal* forces may be defined as such, that when applied to the same mass, generate equal accelerations in the same time.

When a body falls under gravity, its mass remains the same, and its weight *i. e.*, the force on it due to the earth is practical y constant for a short fall. The body is known to fall vertically with a uniform acceleration.

It is also to be noted that, when the mass is constant, a force is proportional to the acceleration communicated to the mass. Thus, as the force exerted by the earth on a body is greater near the pole than near the equator (art. 46), the acceleration due to gravity is also greater nearer the poles.

Thirdly, from the latter part of the law it is understood that, if there are two or more forces acting simultaneously on the same body, each force produces the same effect as if the other forces were not acting. This is a fundamental principle of mechanical science and is known as the **Principle of Physical Independence of Forces**. This gives us, for the composition of two or more forces, the well-known law of the *Parallelogram of Forces*, exactly similar to that of the Parallelogram of Velocities.

38. Unit Force.—In the last article we have seen that P, the force acting on a body at any

instant is proportional to mf, where m is the mass and f, the acceleration at that instant. We may, therefore, put

$$P = k.mf$$

where k is a constant.

If, now the unit force be so chosen, that it produces unit acceleration on unit mass *i.e.*, if $P = 1$, when $m = 1$ and $f = 1$, k becomes equal to 1 and we get

$$P = mf$$

which is known as the *fundamental kinetic equation*.

The unit of force in the F. P. S. system of units is that force which produces an acceleration of 1 ft. per sec. per sec. in a mass of 1 lb. This unit is given the special name of a *Poundal*.

The unit of force in the C. G. S. system of units is that force which produces an acceleration of 1 cm. per sec. per sec. in a mass of one gramme. This unit is called the *Dyne*.

A *Poundal* is a much larger unit than a *Dyne* :—

We have	a pound	= 453·6 gms.
and	a foot	= 30·48 cms.
∴	the poundal	= 453·6 × 30·48 dynes.

39. The Third Law of Motion.—Newton's third law of motion may be stated thus: *To every action there is an equal and opposite reaction ; or the mutual actions of two bodies are always equal (in magnitude) and opposite (in direction.)*

The terms 'action' and 'reaction' in this law apply to forces. Here *action* means the force which one body A exerts on another. The law implies that the second body B always exerts on the first a force in the same straight line, equal to the former in magnitude, and opposite in direction. This force is called the *reaction* of the second body *on* the first. This mutual action between two bodies is generally called a *Stress* ; actions and reactions are thus merely the opposite aspects of the stress between two bodies.

The stress between any two bodies may, however, constitute

(1) a *thrust* or a *pressure*, when each body presses against the other, *e.g.*, the pressure of a book on a table.

(2) A *tension*, when the bodies pull on another through a material medium, as by a string or a rod *e.g.*, the pull exerted by a horse on a cart.

(3) an *attraction* or *repulsion*, when the bodies act on one another *at a distance*, *e.g.*, action of a magnet on a piece of iron.

(4) a *friction* when the stress prevents the bodies from sliding, one along the other.

To quote Newton's own illustration "if a man presses a stone with his finger, his finger also is pressed by the stone ; in other words, the stone resists. Here the pressure exerted by the finger on the stone is an action; and the equal and opposite pressure exerted by the stone on the finger is the reaction."

Similarly, when a book rests on a table, it exerts a pressure (which is its weight), vertically downwards on the table : at the same time, the table resists or exerts pressure vertically upwards on the book. The former is the action, while the latter is the reaction. Since the book is at rest, the forces, *i.e.*, action and reaction must be exactly equal and opposite to each other.

Again, when the ladder is resting against a wall, it presses against the wall which, if not sufficiently strong, may actually be overturned. The wall also exerts an equal force against the ladder, which prevents the ladder from falling over, as it would certainly have done, had there been no support in the same position.

It is important to note that the law holds true whether the bodies are *at rest* or *in motion*.

In walking, when one starts off to walk, he presses backwards on the ground with one of his feet alternately,

and the reaction of the ground gives him an equal and opposite impulse forward, which sets him in motion. The action and reaction in this case are due to friction. On a smooth surface the walking is much more difficult, for a very small amount of friction can be called into play.

It is to be noted, however, that if a body rests on a surface which is *in motion*, for example, a box on the floor of a lift in motion, the action and reaction at the surfaces of contact are, according to Newton's third law, always equal and opposite, but are not *necessarily equal* to the weight of the body. Let W be the weight of the box acting downwards and R the normal reaction of the floor acting upwards. If $R = W$, the resultant force acting on the box is of zero value; and the box and the floor must either be at rest or moving with a uniform motion in a straight line up or down. If, however, the box is subjected to an acceleration f, say a downward one, then (by the second law of motion) the force necessary to cause the acceleration is mf, where m is the mass of the box. In this case the reaction, and hence the pressure of the box on the floor, is *less* than its weight and is $W - R$ which equals mf. The weight of the box may, in fact, be divided into two parts, a force equal to R, exerting pressure on the floor, and the remainder $W - R$, giving the box its downward acceleration. If $R > W$, the floor does not only support the weight of the box, but exerts an additional force $R - W$ which communicates to the box an upward acceleration.

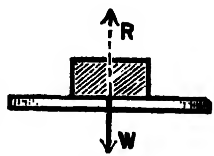


FIG. 28.

When a *horse and a cart* are just starting into motion along a level road, the horse exerts a forward pull or tension, say T, on the cart. It follows from Newton's Third Law of Motion that the cart also exerts an equal and opposite force *i.e.*, a backward pull T on the horse.

If these were the only forces on the system,—regarding the horse and the cart as one system,—no motion of the whole system would have been possible.

But to start motion, the horse exerts pressure against the ground in a backward direction. Again, from the third law of motion it follows that the reaction of the ground on the horse's feet is equal and opposite, *i.e.*, in the forward direction. Let the component of the latter, parallel to the road, be called P. There is another external force R in the form of a resistance due also to the friction opposing the motion of the whole system. So long as P does not exceed R, the system does not move. As the horse exerts more and more muscular effort, P exceeds R, and there is an external force $P-R$ on the system which makes it move in a forward direction.

If we look at the horse and the cart separately, we should say that the horse moves, because P is greater than T, the tension on the horse due to the pull of the cart; and the cart begins to move because T, the pull on it due to the horse, is greater than R.

40. Representation of a Force.—In order to define a force completely, it is necessary to specify

(i) its **Point of Application**, the particular point of a body at which the force is applied. A force may, in general, be supposed to act at any point in its line of action; but in certain cases it is necessary to specify the exact point at which it acts,

(ii) its **Direction**, and

(iii) its **Magnitude**,

All these three particulars can be represented by a straight line, for

(i) one end of the line may represent the point of application of the force;

(ii) the direction of the line gives the direction or line of action of the force. The *sense* of the direction

may be indicated by placing an arrow drawn on the side of the line or on it ; and

(iii) the number of units of length on the line may measure the magnitude of the force.

41. Composition of Forces acting at a Point.— If two forces, say P and Q, act at a point *along the same straight line*, their resultant R is obviously equal to their sum, if they act in the same direction, or equal to their difference, if they act in opposite directions : in the latter case, the resultant acts in the same direction as that of the greater force.

If a number of forces act at a point along the same straight line, the forces acting in one direction may be considered positive and those in the opposite direction negative, then the resultant of the forces is their algebraic sum.

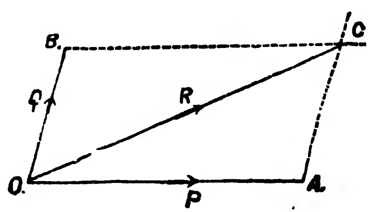


FIG. 29.
Parallelogram of forces.

If two forces, say P and Q, act at a point in directions inclined to each other, their resultant is at once obtained by *the Parallelogram Law.*

The theory of **Parallelogram of Forces** states that :

If two forces acting at a point be represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is

represented both in magnitude and direction by the diagonal of the parallelogram passing through that angular point.

Thus let any two forces P and Q acting at a point O be represented both in magnitude and direction by the two straight lines OA and OB. Complete the parallelogram OACB with OA and OB as adjacent sides. Their resultant R is represented both in magnitude and direction by the diagonal OC passing through O, (Fig. 29).

An application of this rule is met with in the swimming of animals, flight of birds etc. When a bird

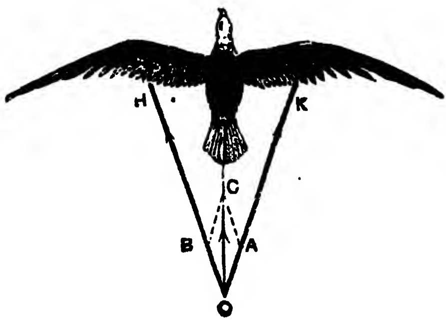


FIG. 30.
Flight of a kite.

flies up in the air, it strikes against the air with its wings. From Newton's Third Law of Motion, it follows that the reactions appear working in an upward direction. If OA and OB represent these reactions fully, and if the parallelogram of force be completed, the resultant force which makes the bird advance, is represented in magnitude and direction by OC. By varying the muscular efforts exerted along AO and BO, the direction CO and hence the direction of flight may be altered.

THE MAGNITUDE OF R :—

The *magnitude* of the resultant may be obtained graphically by measuring the diagonal OC on the same scale on which OA and OB are measured (Fig. 29). It has also been shown in Geometry that

$$OC^2 = OA^2 + OB^2 + 2OA.\ OB\ cos\ AOB$$

Substituting, we have

$$R^2 = P^2 + Q^2 + 2PQ\ cos\ \alpha$$

where α is the angle between the directions of the two forces.

If in a particular case the forces act at right angles to each other, we have

$$\alpha = 90^\circ \text{ and } cos\ 90^\circ = 0,$$

Hence, the expression for R in the formula reduces to

$$R^2 = P^2 + Q^2.$$

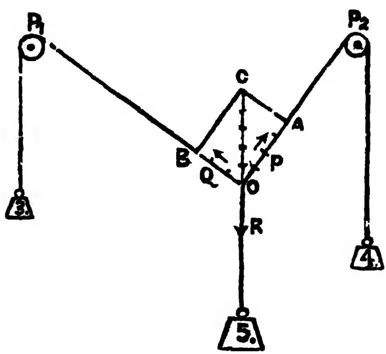


FIG. 31.

PRACTICAL VERIFICATION :—

The truth of the above theorem may be verified experimentally :

Expt. 7. Take three strings and knot them together in a point. Attach to their ends any three weights P, Q and R (any two of which are together greater than the third). Let one string be suspended freely with its weight R, and let the other two pass over two smooth light pulleys P_1 and P_2 placed at the corner tops of a vertical board.

When the suspended system assumes a position of equilibrium, mark on a piece of paper fixed on the board, the directions of P, Q and R. Now take off lengths OA and OB proportional to P and Q respectively. Complete the parallelogram OBCA : join OC.

It will be found that OC is vertical and hence is in the same line with the line of action of the force R ; also that OC contains R units of length in the same scale.

Expt. 8. Spring-balances can be used in place of weights. Attach the ends of three rings knotted at a point to three spring balances L, M and N. Fix the balances to hooks on the edges of

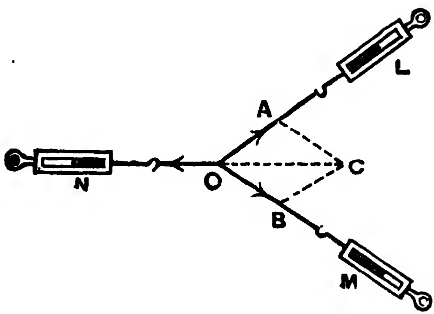


Fig. 32.
To illustrate the Law of Parallelogram of Forces.

a drawing board in such a way that the strings may be kept tight and the balances stretched. The readings of the balances will give the forces acting along the strings, say P, Q and R respectively. Then proceed as in the last experiment.

When any number of forces act at a point in different directions, the magnitude and direction of their resultant may be found by the continued application of the Parallelogram Rule (see also fig. 23).

42. Resolution of Forces—Just as we can compound two or more forces into a single resultant, so conversely we can resolve a single force into a number of others, called its *components*, which are equivalent to it.

To resolve a force into its components in any two given directions, draw a straight line *OC* to represent the given force (see fig. 29). Through *O* draw *OA* parallel to one of these directions and through *C*, draw *CA* parallel to the other. Now complete the parallelogram: then, by the parallelogram rule, the forces *OA*, and *OB* have *OC* for their resultant and are, therefore, the required components (Fig. 29).

As an example of the resolution of forces, we may take the case of a boat sailing partly against the wind. Let *AB* be the direction of the boat's length, *CD* the direction of the sail and *WC* that of the wind. We take *CE* to represent the force, the wind would exert on the sail, if it were placed at right angles to the direction of the wind. Resolve this into two components, one perpendicular to *CD*, and the other along *CD*. The former component *CF* only is efficacious in exerting a pressure on the sail. Now resolve the force *CF* along and across the boat. The component *CK* causes the boat to move forward, while the component *CH* tends to make the boat move through water in a direction perpendicular to its length.

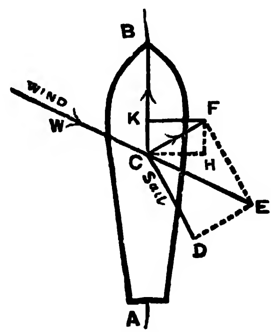


FIG. 33.
The sailing of a boat.

A kite at rest on the air is another example. The kite is acted on by two downward forces; one

is due to its own weight represented by KA and the other due to the pull T along the string. The resultant of these two forces is represented by KC acting downwards. The force of the wind represented by DK on the kite may be resolved into two

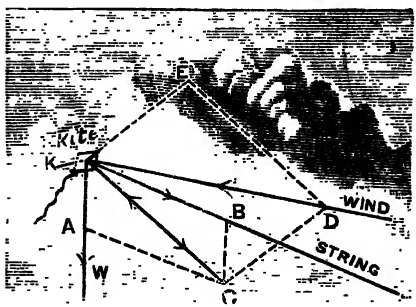


FIG. 34.
Forces acting on a kite at rest in the air.

forces, along and perpendicular to the face of the kite. The latter component is the efficacious one and acts in an upward direction. If this equals the retsultant KC, the kite is at rest ; if it be greater, the kite rises : if less, the kite falls (fig. 34).

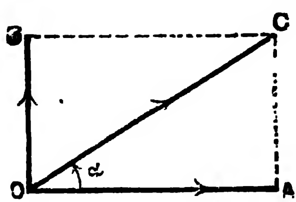


FIG. 35.
Resolution of a force.

The simplest and most important case of the resolution of a force occurs when the force is

resolved into two components at right angles to each other.

Let OC (fig. 35) represent a given force R, OB and OƆ are the two directions at right angles in which the components are required. Let $AOC = \alpha$; draw CA, CB perpendiculars on the two directions. Then OA and OB represent the two required components, say P and Q.

Now $$\frac{OA}{OC} = \cos AOC = \cos \alpha$$

and $$\frac{OB}{OC} = \frac{OA}{CO} = \sin \alpha$$

or $$\frac{P}{R} = \cos \alpha \text{ and } \frac{Q}{R} = \sin \alpha$$

or $$P = R \cos \alpha \text{ and } Q = R \sin \alpha$$

Thus, when a heavy roller is placed on an inclined plane making an angle α with the horizontal

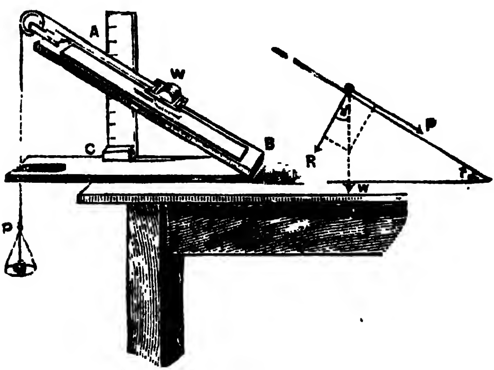


FIG. 36.

A roller on an Inclined Plane.

plane, it rolls down the plane due to its own weight W. It is evident that the full effect of the

force W is not spent in producing motion. Let W be represented by the line OD. Resolve this into a component force P acting in a direction parallel to the plane, and R at right angles to it. The latter produces a pressure against the plane, while the former is the effective force in pulling the roller down the plane. And P is given by

$$P = W \sin \alpha.$$

Expt. 9. Weigh the heavy roller on a spring-balance and find W. Then place the roller on the inclined place and connect it with a pan in the manner shown in fig. 36. Load the pan gradually until the roller is just prevented from sliding down. Evidently, the load on the pan gives the pull which just balances and is, therefore, equal to P. The value of sin α is obtained from

$$\text{Sin } \alpha = \frac{AC}{AB}$$

Hence verify that $P = W \sin \alpha$

CIRCULAR MOTION.

43. Uniform Motion in a Circle ; the Normal Force.

Force.—The only case of the curvilinear motion which we shall consider is that of a uniform motion in a circle.

If a body moves round a circle with a uniform velocity, it passes over equal arcs in equal times. It is to be noted that though the speed remains constant, the direction of velocity is always changing, acting in a tangential direction at any point on the circular path. We have, from art. 33. the relation $\omega = v/r$, where ω is the angular velocity of the body about the centre, v the uniform speed and r, the radius of the circle.

A body once put in motion in any direction tends, owing to its inertia, to move always in that direction with its velocity unchanged. Hence whenever a body is seen to move in a curved path, there

must be some new force urging it at the same time towards the concave side of the curve.

Expt. 10. Tie a piece of stone with a string and swing it round in a circle by holding the other end. It will be felt that in order to keep the stone in its circular path, a force is to be exerted by the hand along the string towards the centre of the path. If the string is cut off or let go, the stone ceases to be acted upon by the tension of the string and tends to fly off in a direction tangential to its circular path at the point where the stone was, when the central force was made to vanish.

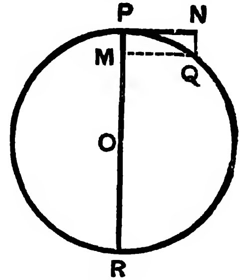


FIG. 37.

The normal force in circular motion

* The intensity of the normal force can be calculated as follows:

Let a body describe a circle of radius r, with a uniform speed v; and let t be the *small* interval of time, during which it passes through PQ, a very small portion of the path. Had it not been acted on by a force in the direction PM, it would have moved through a distance PN in the direction of the tangent at P, where

$$PN = vt \quad \quad (1)$$

Let f be the acceleration along PM. We have

$$NQ = PM = \tfrac{1}{2}ft^2 \quad \quad (2)$$

Squaring (1) and dividing it by (2), we have

$$\frac{PN^2}{PM} = \frac{2v^2}{f} \quad \quad (3)$$

But $PN^2 = MQ^2 = PM.MR = PM.\ 2r$

for PR becomes negligible in comparison with $2r$, when P and Q are consecutive points.

$$\therefore \quad \frac{PN^2}{PM} = 2r = \frac{2v^2}{f}$$

whence $$f = \frac{v^2}{r}$$

Hence when a particle moves in a circle of radius r, with a uniform speed of magnitude v, it is subject

to a constant normal acceleration of magnitude v^2/r directed towards the centre of the circle, at all points in its path

$$\text{And the corresponding normal force} = \frac{mv^2}{r}$$

where m is the mass of the body.

44. Centripetal and Centrifugal Forces.—The deflecting force, spoken of in the last article, which acts *upon* the rotating body in a direction *towards* the centre of curvature in order to constrain it to move in a curved path, is called the *Centripetal Force.* Its magnitute is given by mv^2/r (art. 43).

Since every action is accompanied by a reaction, equal and opposite to it (Newton's third law of motion), there is also a force that is *directed away from the centre* ; this force is called the *Centrifugal force.* It is a force exerted *by* the rotating body *on* the centre of curvature Thus in the case of the stone in the sling in Expt. 10 it is the force exerted *by* the stone *upon* the string and *not* a force acting upon the stone. It is due to this force that the string is kept tight against the centre of curvature. It is to be noted that if, in any case, the centripetal force vanishes, its reaction *viz.*, the centrifugal force does also vanish at once.

Centrifugal force.

When we look at the stone in the sling, it is clear that it has a rectilinear velocity along the tangent to any point in its circular path and a normal acceleration due to the centripetal force acting upon it, the result of which is that it is constrained to move in the circular path. If at any moment the string be cut off at any point P in its path, or the hand at the centre ceases to exert a pull towards it, the centripetal force does no more exist ; and the stone due to its inertia flies off simply with the linear velocity which it had at the instant under consideration, and goes in a direction PN (fig 37)

Stone in the sling.

tangential to its curved path at P. The stone thus goes from P to N when it would have otherwise gone from P to Q, which makes it *appear to fly* from the centre.

Illustrations of the influence of centripetal force are ample. The road-bed at a bend in a railway line is often inclined, so that the outer rail is a little higher than the inner ; when a train rounds a curve, a part of its weight and the reaction at the flange supply the centripetal force necessary to keep the train in the curved path. Moreover, the action is helped by reducing the speed of the train at the bend.

For a similar reason, a horse and its rider in a circus always incline their bodies inward, and the greater their speed, the greater is this inclination. In the same way, a man riding a bicycle feels it impossible to take a bend in his path without slackening the speed or leaning a little towards the bend

Again, a planet is kept in its orbit round the sun, which is almost circular, by its velocity acting in a direction tangential to its path at any point, and the attractive force of the sun on it.

In looping the loop, or in the toy, called the *Centrifugal Railway*, the velocity acquired by the car in fall-

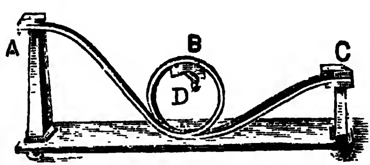


FIG. 38.

The Centrifugal Railway.

ing through a height and the reaction at each point of the loop exerted against the car enable it to round the bend. Similarly, if a small can of water hung by the handle is rapidly swung round in a vertical circle, no water will come out, even when the bottom is uppermost.

It is owing to the acquired linear velocity and inertia again that the mud particles which stick to the wheels of a carriage fly off when the wheels are rapidly rotating.

The whirling table in a lecture-room affords many experiments exhibiting the presence of a centripetal force for a circular motion :—

Expt. 11. Two balls slide freely along a brass rod, supported in a brass frame AB. Fix the frame on a turning table. If either of the balls be not exactly at the centre of rotation, the two balls in the absence of any centripetal force fly off along the rod due to the linear velocity they acquire; and they strike the side with a great force which increases with the velocity of rotation (fig. 39)

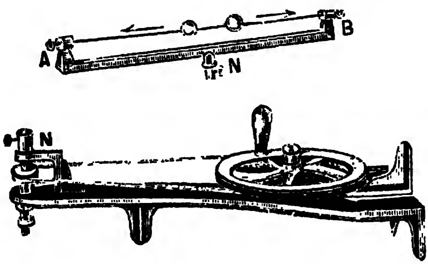


Fig. 39.
The rotating table.

Fig. 40 represents a portion of *Watt's Governor* which is used to regulate the supply of steam in some forms of steam-engines. Two heavy balls are fastened to arms hinged at the top on a vertical axis which is connected with the main shaft of the engine to share its rotation. Mechanism for opening and closing the steam-ports so as to vary the supply of steam is connected with the balls. When the engine runs too fast, the balls rise and partially cut off the supply of steam; when the speed is too slow, the balls fall and open the port. In this way, the engine automatically controls its own speed.

Expt. 12. Fit the framework on the whirling-table. See that when the speed of rotation increases, the balls fly outwards, while they fall and approach the axis when the speed diminishes.

Fig. 41 represents a model to illustrate the flattening of the earth by rotation. It consists of a central rod that carries several thin elastic strips

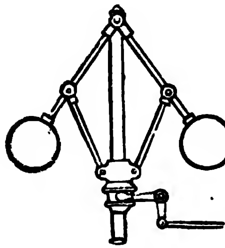


FIG. 40.

FIG. 41.

of metal in the form of a circular hoop. The hoop is fixed at the bottom and is attached to a collar at the top which can slide up and down the rod.

Expt. 13. Fix the apparatus on the whirling-table and put it in rapid rotation. Each particle of the elastic strips tries to move outward from the axis with the linear velocity it acquires. This is allowed by the collar sliding down the rod to an extent depending on the rapidity of rotation and elasticity of the spring. Thus the hoop is bent from its circular form and becomes flattened at the top and the bottom, and bulged at the centre.

The earth was, as is held by the geologists, once in a hot state of fusion when its materials yielded to the effects of rotation like the elastic strips of the hoop in the experiment. And the earth has become a spheroid having a bulging at the equator and a flattening at the poles, so that its equatorial diameter is 7925 miles, and the polar one is 7899 miles. A similar flattening has been observed in other planets too.

The apparent tendency to fly off from the centre of bodies set in rotation has been used in chemical laboratories to separate crystals from the mother-liquors; in dairies, to separate cream from the heavier milk; it is also applied industrially in sugar factories to purify sugar from syrup, to dry yarns in dye-works and clothes in large laundries.

45. The Apparent Weight of a Body.—Bodies on the surface of the earth, while they share the diurnal motion of the earth, seem to lose a portion of their true weight. The weight of a body, as is seen before, is due to the earth's gravitative force which tends to attract the body towards its centre. Let W be this true weight of a body As the body is sharing the rotation of the surface of the earth where it is, a part of its weight must act as the centripetal force necessary to prevent the body from flying off; the remainder is the so-called weight of the body, which may be called its *Apparent Weight.* Hence

$$\text{the apparent weight of a body} = W - \frac{mv^2}{r} = mg \text{ .}$$

where v is the velocity of the part of the earth's surface where the body is, and r is the radius of the circle described by the surface round the earth's axis of rotation.

46. Increase in Weight of a Body from the Equator to the Poles—It is evident that if a body be taken at the equatorial region of the earth, it describes in the same time a circle of larger circumference than that of a body situated in the polar regions. As the angular velocity is the same, the part of the weight of a body, which acts as the centripetal force necessary to prevent a body from flying off and which is given by mv^2/r must be greater in the former case. Hence the weight of a body increases as it is taken nearer to the poles.

The peculiarity in the earth's shape also concurs in producing the same effect. In consequence of the flattening of the earth at the poles, bodies on the surface of the earth at the poles are $13\frac{1}{2}$ miles nearer the centre than those in the tropical regions, and are, therefore, attracted with a greater pull, for the force of attraction varies inversely as the square of the distance (Art. 105).

This difference in weight can not, however, be detected by an ordinary balance, for the change in the

earth's attractive pull will equally affect the standard weights and the body to be weighed. A delicate spring-balance (art. 11) is to be used for this purpose.

47. Moment of a Force.—If a body be so situated that a line in it is kept fixed, so that the body can not move out of its own place (*translation*), but is free to rotate about that line, the force applied to it in any direction not passing through the line will cause *rotation*.

The rotatory effect of a force depends on

(i) the magnitude of the force, and

(ii) the perpendicular distance of its line of action from the fixed axis, generally called the *Axis of moment*.

Consider, for example, a door turning about its hinges. It requires a much smaller force to open or shut the door, if the force be applied in a direction perpendicular to the door, and at a considerable distance from the hinge, than if applied near to the hinge.

The tendency of a force to produce rotation about a fixed axis is called its *moment*, and is measured by the product of the magnitude of the force and the length of the perpendicular from the axis to the line of action of the force. Thus if F_1 be the force acting on a rigid body capable of rotating about an axis through O perpendicular to the paper, and OA be drawn perpendicular to the line of action of F_1,

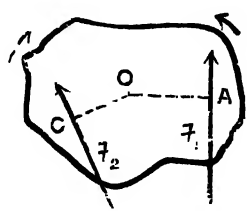


FIG. 42.
Moment of a force

then the moment of F_1 = Force × arm = F_1 × OA.

Hence the moment of a force about a point vanishes when either

(i) the force itself is zero, *i.e.*, $F_1 = 0$

or (ii) the line of action of the force passes through the point *i.e*, the arm = o.

Further, the moment of a force is regarded ***positive***, if it tends to produce rotation in the anti-clockwise direction, and ***negative*** if it tends to produce rotation in the same direction as the hands of a watch go. Thus in fig. 42 the moment of F_1 about O is $+ve$, while that of F_2 about O is $-ve$.

It is evident from the above that if a rigid body, movable about a fixed point, is kept in equilibrium by two forces in any plane containing that point, the moments of these forces about the point will be equal and opposite.

In general, if a body be at rest under the action of several forces in the same plane, the total moment of all the forces about every point in the plane is zero, *i.e.*, the clockwise moments are equal to the contra-clockwise moments. This principle is of great use in finding the amount or direction of some unknown force on a body which is at rest.

Principle of Moments.

48. Composition of Parallel Forces.—Forces, whose lines of action are parallel, are called the *Parallel Forces*. They are of frequent occurrence in practical mechanics

Parallel forces acting in the same direction are said to be ***like***, and acting in opposite directions are said to be ***unlike***.

The resultant of two *like* parallel forces P, Q, acting at A and B respectively, must evidently be equal to their sum, *i.e.*, $R = P + Q$, and must act in the same direction as the forces, the line of action of the resulant lying between those of the component forces. Its point of action G is on the line AB, and may be proved to be given by

$$P + AG = Q + BG$$

Or $$\frac{P}{Q} = \frac{BG}{AG}$$

In other words, the point G divides the line AB *internally* in the inverse ratio of the forces.

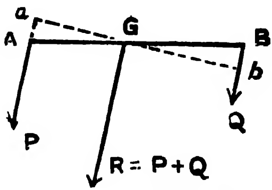


FIG. 43.

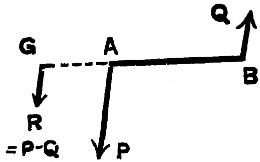


FIG. 44.

Composition of parallel forces.

The resultant of two *unlike* parallel forces P,Q, acting at A and B respectively, is equal to their differences, *i.e.*, $R=P-Q$ (assuming P to be greater than Q), and acts in the sense of the greater force P. It may be proved that it acts at a point which divides AB *externally* into parts which are in the inverse ratio of the forces, *i.e.*,

$$P \times AG = Q \times BG.$$

It follows that the resultant is, in both the cases, nearer to the greater force.

In both the cases, draw a line aGb through G so as to be perpendicular to the line of action of the parallel forces, so that a and b are on the lines of action of P and Q respectively. Then from similar triangles AGa and BGb, we have

$$\frac{AG}{BG} = \frac{aG}{bG}$$

or
$$\frac{P}{Q} = \frac{bG}{aG}$$

or
$$P \times aG = Q \times bG,$$

i. e. the moments of P and Q about G are equal and opposite.

Expt. 14. Take a rectangular bar of wood A, and fix a metre scale on one of its faces. Suspend it from a fixed support

by means of a spring-balance connected with the hook D passing through a hole in the bar.

B and C are rings of wire, carrying small hooks below, and can slide along the bar. Support weights from B and C, and adjust them so as to make the bar horizontal.

Let the total weight supported at $B = P$ kilos.
and ,, ,, ,, ,, ,, ,, $C = Q$ kilos.

Let the reading in the spring-balance $= R$ kilos.

It will be seen that

$$R = P + Q$$

and also that

$$P \times BD = Q \times CD.$$

The resultant of P and Q is evidently a force through D, equal and opposite to R.

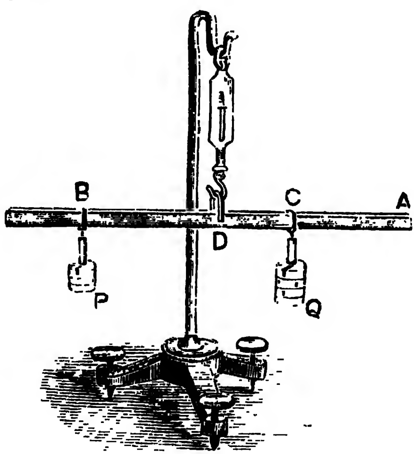


FIG. 45.

Composition of parallel forces.

Shift the position of D. This requires the ring C also to be shifted so as to keep the bar horizontal. It is to be noted that the sum of the weights P and Q remains constant and equal to R, and that the condition

$$P \times BD = Q \times CD$$

is always satisfied.

Again, of the three forces P, Q and R in Expt. 14, if P and R be regarded as the two component unlike forces, Q may be looked upon as a force, equal and opposite to the resultant of R and P, such that $R-P=Q$. It will also be seen that the position of C is such that $P\times BC=R\times CD$.

49. Couple—In the previous article if the two unlike parallel forces be equal in magnitude, the resultant of P and Q is zero. So no translatory motion is possible. The couple may have, however, a moment about any point so as to produce rotation.

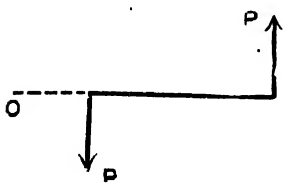


FIG. 46.
Moment of Couple.

Let O be a point either between P and P, or outside them. Draw OAP perpendicular to the forces. Then the algebraic sum of the moments of two forces P and P about O

$$= P.\ OB \pm P.\ OA$$
$$= P.\ (OB \pm OA) = P.\ AB = \text{force} \times \text{arm}.$$

Hence the *moment of the couple* about O is independent of the position of O, and vanishes if either a force of the couple or the arm vanishes.

As an example of couple action we may cite the case of the earth's magnetic action on a magnetic needle. If a needle be displaced from the magnetic meridian, it quickly returns to it indicating thereby that the earth's action on it is of the nature of a couple.

Excercise – IV

1. What is the Dyne? Show that it is nearly equal to the weight of a milligram.

2. State and exemplify the Third Law of Motion.

3. Find the resultants of the following forces, the angle within them being given :—

(*a*) 8 and 10 gm ; 0° (*c*) 17 and 144 kg ; 90°

(*b*) 12 and 24 oz ; 120° (*d*) 7 and 8 oz ; 150°

4. The greatest resultant of two forces is 31 lbs and the least is 17 lbs ; what is the resultant when the two forces act at right angles to one another ?

5. A boy pulls a loaded truck weighing 200 lbs. up a hill of 1 in 5. Neglecting friction, what force must be exerted ?

6. A ball of mass, 3 lbs, is falling at the rate of 100 ft. per sec. What force in pounds' weight will stop it (i) in 2 sec. (ii) in 2 feet ?

7. Forces 6 lbs, 8 lbs, and 10 lbs act at a point in directions parallel to the sides of an equilateral triangle taken in order. Find their resultant.

8. A steam-engine is moving at the rate of 18 miles an hour, when the steam is shut off. If the force of friction be equivalent to $\frac{1}{283}$ of the weight of the engine, find the time after which it will stop.

9. A man carries a bundle at the end of a stick over his shoulder. If the distance between his hand and the bundle be kept constant, and the distance between his hand and shoulder be varied, how does the force on his shoulder change ?

10 Two men carry a load of 1 cwt. suspended from a horizontal uniform pole 12 ft long, whose weight is 20 lbs. and whose ends rest on their shoulders. Where must the load be suspended in order that one of them may bear 94 lbs. of the whole weight ?

11. A uniform rod, 4 ft. long and weighing 16 lbs. rests on a horizontal table with one end projecting 8 inches over the edge. Find the greatest weight which can be hung on the end without making the rod topple over.

12. State Newton's Second Law of Motion, and explain how it enables you to measure forces. Is weight an essential property of matter ? How do you distinguish between mass and weight ? If the weight of a gram be taken as the unit of force, what is the unit of mass ? [*Pat. U.—1918.*

CHAPTER V.

FALLING BODIES

50. Force of Gravity on Falling Bodies:—It is a matter of every-day experience that whenever a heavy body is unsupported, it falls to the ground. And as it falls, its velocity is gradually increased ; in other words, the motion is accelerated.

Since there is an acceleration or a change of velocity in the downward direction, it follows from Newton's Second Law of Motion that there is a force in this direction. This attractive force of the earth is called **Gravity**, and the acceleration produced by it is called the *acceleration due to gravity*, or simply the *acceleration of gravity* and is always denoted by the letter g.

The action of gravity is an illustration of and at the same time a particular case of the *Universal Gravitation* (art 105), the laws whereof have been enumerated by Newton. From these laws it follows that the force exerted by the earth on a body,—in other words,—its weight depends upon

(1) the mass of the body (m).
(2) the mass of the earth (M)
and (3) the distance of the body from the centre of the earth (R).

When a body falls through a small height, as is ordinarily the case, its distance from the centre of the earth remains practically the same. Hence the weight of a body due to which it falls downwards, is ordinarily a constant force exerted on the body at the same place on the earth's surface. Experiment with a delicate spring-balance shows no change in the weight of a body, if it be raised or lowered through a few yards.

Further, as M, the mass of the earth does not change, and the change of R, as has been said above, is not appreciable in the ordinary upward or downward displacement of a body, the weights of different bodies are proportional to their masses.

51. Acceleration of Falling Bodies is the same for all Bodies.—Since the weights of different bodies at the same place are proportional to their masses, we may put that W/m is constant.

Again when a body falls freely under the force of gravity, the kinetic equation $P=mf$ in the Second Law of Motion becomes $W=mg$, whence $W/m=g$. It follows that g is constant. In other words, all bodies when unsupported, fall towards the ground with the same acceleration, hence with equal rapidity.

But this is not in conformity with our common experience, as we are accustomed to see light bodies such as feather, pieces of paper, etc. fall very slowly to the ground, while a heavy body let fall from the same height touches the ground very quickly. The difference in the rapidity of fall was formerly thought to be inherent in the nature of the materials of bodies.

Previous to GALILEO's time men believed in ARISTOTLE (about 3?7 B C.) who had said that a ten-pound weight would fall ten times as fast as a one-pound weight.

Galileo's experiment on falling bodies (1950).

Galileo in the latter half of the sixteenth century argued that two such weights, dropped from the same heights, would take exactly the same time in falling to the ground. In the presence of the professors and students of the University of Pisa he dropped balls of different sizes and materials from the top of the *Leaning Tower of Pisa*, 180 ft. high. and showed that they fell in precisely the same time. He held that all bodies even the lightest would fall at the same rate, but for the resistance offered by the air to impede the motion of bodies, the resistance increasing with the extent

of surface exposed by the body. Thus the effect is more marked, the lesser the mass of a body and the greater the surface it presents. If, by some means or other, this air resistance be eliminated, all bodies, heavy or light, would take the same time to fall through a given height. Modern Physics may be said to begin with Galilio who discovered the laws of falling bodies and the laws of the pendulum.

Galileo's conclusion that in a vacuum all bodies would fall with the same velocity,—could not, however, be put to test by experiment in his time, the air-pump not having yet been invented. After its inventions sixty years later (1650) by OTTO VON GUERICKE, the experiment was performed by NEWTON and is now well known as the '*Guinea and Feather*' experiment.

Expt. 15. Take a large coin or a disc of metal of about 2″ diameter and cut a piece of paper slightly smaller than the coin. Hold these side by side at the same height above the table and drop them simultaneously. The metal disc touches the table first and then does the paper.

Now lay the paper on the top of the metal and drop the whole carefully. Both touch the table simultaneously. Here the metal disc overcomes the resistance of the air which would otherwise retard the motion of the paper.

Repeat the experiment by placing the paper on the metal disc such that part of its surface is exposed to air. Now the paper will be left behind but not to the same extent as in the first case.

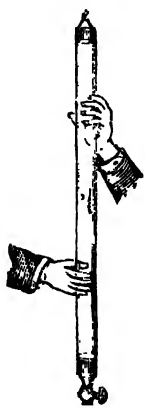


FIG. 47.
Guinea and Feather Experiment.

Expt 16. Fig. 47 represents a stout glass tube about a metre long, closed by a cap screwed to one end, and is provided with a stop-cock tied to the other. Introduce a coin and a piece of paper or a feather into the tube. Exhaust the inside of the tube by means of an air-pump. After closing the stop-cock, disconnect the tube from the pump.

Invert the tube suddenly; the coin and the paper will strike the bottom simultaneously. Repeat this.

Now introduce a little air within the tube by opening the stop-cock for a moment. Invert the tube again: now the feather becomes slightly retarded in its fall.

It will be an interesting point to note the manner in which liquids fall in a space devoid of air. The water-hammer (fig. 48), illustrates this. This is a thick glass tube, about a foot long, which is half full of water, all the air having been removed by boiling before the tube was sealed at one end with a blow-pipe. When such a tube is given sudden jerks, the water strikes either end with a blow, like that of a hammer, producing a sharp, dry, metallic sound.

FIG. 48.
Water-hammer

52. Acceleration of a Falling Body is Constant.—When a heavy body originally at rest is allowed to fall, it is acted on during this time of its descent by its own weight and by no other force, if the resistance of air be neglected. Now as the weight of a body is a constant force, we infer from the relation $W = mg$ that g, the acceleration is constant for the same body and for the same place.

Determination of g.

The most direct way of proving this practically would be the observation of the distances, a body falls through, in various intervals of time and then testing whether the relation $s = \frac{1}{2}gt^2$ holds or not. But this method is not ordinarily easy to put into practice: for unless the intervals of time are very short, the space transversed and the velocity acquired soon become considerable and difficult to measure; again, if the times are short,* it is difficult to measure them exactly.

* A *tuning-fork* may be used to mark small intervals of time for the above purpose.

Expt. 17. In fig. 49 T is a large tuning-fork, making say 20 vibrations per second. It is provided with a light bristle in the end of one of its prongs and mounted in such a way that the prongs vibrate in a horizontal plane. A long strip of plate-glass *G* is supported in an upright, such that its lower part is touched by the bristle end. The front of the plate is covered with lamp-black and the sides work in grooves which guide the plate in its

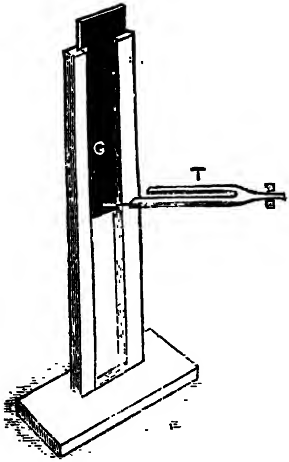


FIG. 49. FIG. 50.

Determination of *g* by the 'Falling Plate' method.

fall. An india-rubber pad is fitted at the base of the groove so as to receive the plate at the end of its fall.

Excite the fork and allow the plate to fall, the fork traces a sinuous curve on the plate (fig. 50).

Draw the middle line AF in the curve obtained. If A be the starting point of the trace, it will be seen, on measurement with a pair of dividers and a diagonal scale, that approximately

$$AC = 4\,AB = 2^2.\ AB.$$
$$AD = 9\,AB = 3^2.\ AB.$$
$$AE = 16\,AB = 4^2.\ AB.$$

i. e., the spaces traversed in 2, 3, 4 etc., units of time (the unit of time being here one-twentieth of a second) are proportional to the squares of the time. But this is only possible when a particle moves with uniform acceleration. Hence we infer that the accele-tion due to gravity is uniform.

Sometimes AB will be too small to measure conveniently. In that case, measure any two consecutive distances d_1 and d_2 in two consecutive equal times t. Then, if the acceleration is uniform,

d_1/t = the velocity of the plate at the middle of the 1st interval.

and d_2/t = the velocity of the plate at the middle of the 2nd interval.

$\therefore$ $(d_2 - d_1)/t$ = change of velocity in the interval between the middle of the 1st interval and that of the 2nd interval.

= change of velocity in time t.

Hence the acceleration, if uniform $= (d_2 - d_1)/t^2$.

Calculating from various parts of the trace, it may be shown that the same value for the acceleration is obtained from all, and that this approximately is

981 cm. per sec per sec.

53. Galileo's Experiment.—To avoid the difficulty of measuring the spaces traversed by a freely falling body, which soon becomes very large, GALILEO used an inclined plane to slacken the motion of the body so as to render it observable. He conducted some experiments at Pisa about the year 1590. He constructed a smooth groove on a plane surface which could be inclined at any angle, and used a polished brazen ball as the falling body.

Galileo made a series of marks down the groove at distances proportional to 1, 4, 9, 16 etc, from the starting point, and found that these marks were passed by the ball at times represented by 1, 2, 3, 4 etc. Thus the

distances traversed from rest were proportional to the squares of the times, whence it follows that the acceleration down the plane is constant. Again the acceleration down the plane due to the earth's attraction is a constant fraction of g, the acceleration in a vertical direction,—in fact it is given by $g \sin \theta$, where θ is the angle of inclination of the plane (see art. 43). It follows that 'g' is also constant.

54. The Value of 'g'.—The value of g may be approximately determined by the Falling Plate method, by the Inclined Plane method or by Atwood's Machine. It is accurately determined, however, by means of the experiments with the Pendulum (art. 88).

It has been established in the previous article that the value of g, the acceleration due to the attraction of the earth, is *constant at the same place.*

Secondly, in consequence of the flattening of the earth at the Poles and the bulging at the Equator, the attraction due to gravity and hence the accleration g are less at the equatorial regions. The value of g varies with the latitude of a place, increasing from the equator to the poles. Thus at the sea-level

at equator,	$g=32.09$ ft.	or 978·10 cm.	per sec. per sec.
at Paris,	$g=32.18$ ft.	or 980·94 cm.	" "
at London.	$g=32.19$ ft.	or 981·17 cm.	" "
at Calcutta.	$g=32.13$ ft.	or 978·76 cm.	" "
at Poles,	$g=32.25$ ft.	or 983·11 cm.	" "

As an average value of g, we may remember that

$g=32$ ft. per sec. per sec., or 981 cm. per sec. per sec.

Again, the earth's attractive force, and hence g too depend on the altitude, each being greatest at the sea-level and less at the summit of a high mountain.*

55. Unit of Weight.—It has already been said that the weight of a body at any place on the earth's

*See also art. 88.

surface is the *force* with which the earth attracts the body towards it. This force acts towards the centre of the earth, and its direction at any place determines the vertical direction at that place.

Expt. 18. Take a plumb-line and test by means of it whether an upright, say a telescope stand, is vertical or not.

Note that a balance is always provided with a plumb-line for levelling purpose. Level a balance by adjusting the foot-screws.

If we apply the formula $P = mf$ in the case of a body falling freely at any place, we may write $W = mg$, where W is a particular force, called the weight of the body, and g the acceleration due to gravity. W is here expressed in the absolute unit of forces, either the poundal or the dyne according as the mass taken is one pound or one gramme.

Care must be taken to make a distinction between the *weight of a pound* and a *poundal*. The **Poundal** is a force which acting on one pound produces an acceleration of 1 ft. per sec. per sec. The weight of a pound or a **Pound-weight** is also a particular force but different from the poundal. It is the force which the earth exerts on a mass of one pound and generates in it an acceleration of g or 32 feet. per sec. per sec. and is evidently equal to 1×32 *poundals*. For the sake of convenience, a *pound-weight* is chosen as the practical unit of force (**Gravitational unit**). But as the value of g varies from place to place, this unit cannot be constant.

Similarly, the weight of a gramme at a place where the acceleration due to gravity is 980·8 cms. per sec. per sec. is 980·8 dynes.

56. Laws of Falling Bodies.—As a body, when dropped from a height falls freely with an uniform acceleration g, all problems concerning falling bodies may be solved by the formulæ established in art. 25 in which g is to be substituted for f, the resistance of the air being neglected.

If a body start from rest, being simply dropped from a height, we have for its velocity at time t

$$v = gt \quad \ldots \quad (1)$$

for the space traversed

$$h = \tfrac{1}{2}gt^2 \quad \ldots \quad (2)$$

From (1) & (2) $\quad v^2 = 2gh \quad \ldots \quad (3)$

From these equations and from art. 51 we may state the following laws of falling bodies in vacuo, *i.e.*, when they experience no resistance *viz.*,

(1) In a vacuum all bodies fall with equal rapidity. This law has been established in art 51.

(2) The velocity acquired by a falling body is proportional to the duration of its fall *i.e.*, $v \propto t$. Then if the velocity at the end of a second is 32 . per sec., that at the end of two seconds is 64 ft. per sec., at the end of 3 seconds is 96 ft. per sec., and so on.

(3) The space traversed by a falling body in a given time is proportional to the square of that time, *i.e.*, $h \propto t^2$. Thus if a stone falls through 16 ft., in one second, it will drop through 64 ft. in two seconds, 144 ft. in 3 seconds and so on.

Since a body, when allowed to fall freely under the action of gravity, soon acquires a velocity which becomes too great to permit of measurement, arrangements have been devised to reduce this to a measureable amount. The Inclined Plane (art. 53) and Atwood's machine (invented about the year 1784) are used to verify the last two laws.

57. Fall of Bodies Projected downwards or upwards,—If a body be projected downwards with an initial velocity u, the formulæ of art. 25 become

$$v = u + gt \quad \ldots \quad (1)$$

$$h = ut + \tfrac{1}{2}gt^2 \quad \ldots \quad (2)$$

$$v^2 = u^2 + 2gh \quad \ldots \quad (3)$$

If a body be projected upwards with a given upward velocity u, we are to substitute $-g$ for f, since acceleration due to gravity is opposing, the upward direction of u being taken as positive. The formulæ now become

$$v = u - gt \quad \ldots \quad (4)$$

$$h = ut - \tfrac{1}{2}gt^2 \quad \ldots \quad (5)$$

$$v^2 = u^2 - 2gh \quad \ldots \quad (6)$$

From these formulæ any problem on the motion of falling bodies may be solved. For example, the greatest height h to which a body rises when projected upwards with a velocity u, is given by

$$0 = u^2 - 2gh$$

for $v = 0$, when the body just ceases rising.

$$\therefore \quad h = u^2/2g$$

Similarly, the time to attain this greatest height is given by

$$v = 0 = u - gt$$

$$\therefore \quad t = u/g$$

Again, after reaching its greatest height the body will begin to fall and when it returns to its point of projection, the final height becomes zero. The whole time of flight may be obtained by putting $h = 0$ and $f = -g$; we thus have

$$0 = ut - \tfrac{1}{2}gt^2$$

$$= t\,(u - \tfrac{1}{2}gt)$$

Discarding the solution $t = 0$, we get

$$u = \tfrac{1}{2}gt \qquad \text{or} \qquad t = 2u/g.$$

EXAMPLES :—

1. **Draw a curve, on the squared paper supplied to indicate the height above ground, at intervals of half of a second, of a body falling freely from rest at a height of 150 ft. Find from your graph the position of the particle after 1·67 seconds.** [*C. U.I.Sc.—1912.*

The space traversed by a body in time t falling from rest can be obtained from the formula

$$h = \tfrac{1}{2}gt^2$$

Taking $g = 32$ *ft. per sec per sec.* and calculating the distances allen through at the end of every half second, the following table has been prepared :

TIME IN SECONDS.	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
HEIGHT FALLEN THROUGH.	4	16	36	64	100	144
HEIGHT ABOVE GROUND IN FEET.	146	134	114	96	50	6

In the graph given below

1 small div. along X-axix = 0·1 sec.

1 small div. along Y-axis = 5 ft. per sec.

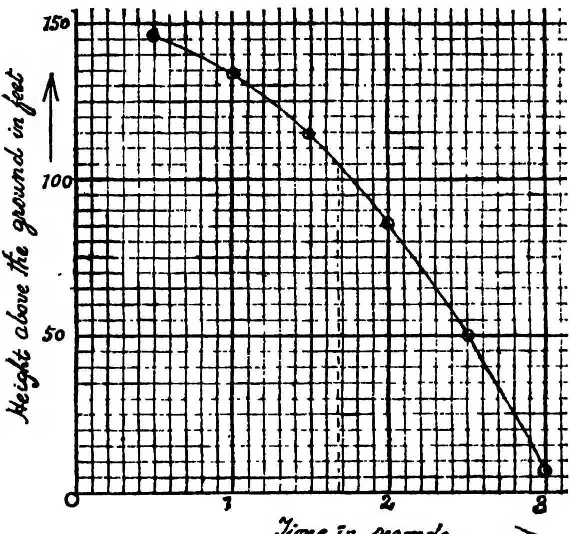


FIG. 51

Fall of a body from rest.

2. A steam-engine starts on a downward incline of 1 in 100 with a velocity of $7\frac{1}{2}$ miles an hour; neglecting friction, find the distance traversed in 1 minute.

Acceleration down the inclined plane

$$=g \sin \theta = g\frac{h}{l} = 32 \times \frac{1}{100} = \frac{8}{25}$$

Again $7\frac{1}{2}$ miles an hour $=11$ feet per second.

$S = ut + \frac{1}{2}ft^2$ gives

$S = (11 \times 60 + \frac{1}{2} \cdot \frac{8}{25} \cdot 60 \times 60)$ feet.

$= 2236$ ft. $= 412$ yards.

3. If a heavy body is thrown vertically up to given height, and then falls back to the earth, show that, neglecting the resistance of the air, it passes each point of its path with the same velocity when rising and when falling.

Let u = velocity of projection

v = vel. at a given height h

Then $v^2 = u^2 - 2gh$

$$v = \pm \sqrt{u^2 - 2gh}$$

which shows that the velocities are equal numerically, but opposite in sign, at any point of the path, when rising and when falling.

4. A ball is thrown up, and caught again in 6 seconds. Find the velocity of projection, and the greatest height.

Let u = initial velocity and h = greatest height.

From $s = ut + \frac{1}{2}ft^2$ we get here

$o = u.\ 6 - \frac{1}{2}.\ 32.\ 6^2$

whence u = 96 ft. per sec.

And $v^2 = u^2 - 2fs$ gives

$u^2 = 2gh$ or $h = \frac{96 \times 96}{2 \times 32} = 144$ ft.

Exercise.—V.

1 A body falls freely for 6 seconds; through what distance will it fall in the last second and in the whole time?

2. Distinguish between mass and weight.

If the weight of a certain mass be represented by 15 at a place where a body falls through 64 ft. in 2 seconds, what will be the weight of the same mass at a place where a body falls through 176 ft. in 3 seconds ?

3. A cannon-ball is shot horizontally from the top of a tower 49 ft high, with a velocity of 200 ft. per seconds. Find at what distance from the tower the cannon ball will strike the ground ?

4. A stone is let fall from the top of a railway-carriage which is travelling at the rate of 30 miles an hour. Find what horizontal distance and what vertical distance the stone will have passed through in one-tenth of a second.

5. The intensity of gravity at the surface of the planet Jupiter being about 2·6 miles as great as it is at the surface of the earth, find approximately the time which a heavy body would occupy in falling from a height of 167 feet to the surface of Jupiter.

6. An arrow is shot verically upwards with velocity of 104 feet per second when it leaves the bow. How long will it be before it reaches the ground again ?

7. Suppose that at the equator a straight hollow tube was thrust vertically down towards the centre of the earth, and that a heavy body was dropped through the centre of such a tube. It would soon strike one side ; find which, giving a reason for your reply.

8. A stone is dropped from a balloon at a height of 100 ft. above the ground. and it reaches the ground in 6 seconds. Find the velocity with which the balloon was rising.

9. A stone dropped into a well reaches the water with a velocity of 80 ft. per second, and the sound of its striking the water is heard $2\frac{7}{21}$ seconds after it is let fall. Find from these data the velocity of sound in air.

10. Explain what is meant by *mass* and *weight* and how they are measured.

A body is weighed at the surface of the earth at sea-level and at the top of a mountain. State, in general terms. how the position will affect the weight and the mass of the body. Give reasons for your answer. [*C. U.—1920.*

CHAPTER VI.

CENTRE OF GRAVITY.

* **58. Centre of Parallel Forces.**—Let a number of like parallel forces of magnitude P, Q, R, S, etc., act at points A, B, C, D, etc, of a body (fig. 52). The resultant force of P and Q is a force of magnitude $(P+Q)$ acting at a point E in AB, such that $P \times AE = Q \times EB$, and is parallel to both of them. The forces $(P+Q)$ acting at E and R at C are equivalent to a single resultant force $(P+Q+R)$ acting at F, parallel to them such that $(P+Q)$ $EF = R \times CF$.

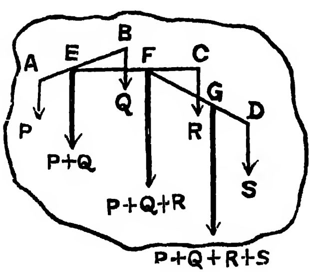


Fig. 52.

Centre of Parallel Forces.

Similarly, the resulant of $(P+Q+R)$ acting at F, and the force S at D is the force $(P+Q+R+S)$ acting at a point G such that

$$(P+R+Q)\ FG = S \times GD.$$

Proceeding in the same way we may find the resultant of any number of parallel forces acting at different points of a body..

If the forces P, Q, R, S etc., applied at the same points A, B, C, D, *etc.*, act in a different direction, remaining still parallel to one another, their resultant will still pass through the same point G as before.

The point G is called the **Centre** of the parallel forces. It is a point through which the resulant of any number of parallel forces passes, its position

depending only on the magnitudes of the forces and the positions of the points at which they act, and is quite independent of their direction.

59. Centre of Gravity.—We have already seen that the weight of a body is the force with which the body is attracted by the earth towards its centre, the direction of action of this force being called the *vertical* direction at the place where the body is.

Now we may consider a body to be made up of a large number of small heavy particles rigidly connected together, and that each of these particles is acted on by its weight, proportional to its mass. These forces all act towards the earth's centre, but having regard to to the long distance, nearly 4000 miles, of the body from the earth's centre, and the body being small compared to the earth, the forces may be considered parallel. The weight of a body, as a whole, is then really the resultant of a system of parallel forces, made up of the weights of all the particles which build up the body, acting at points where these particles are situated. The point at which this resultant acts is called the **Centre of Gravity** of the body.

In art. 58 it is seen that the position of the centre of a system of parallel forces is independent of the direction of the forces. Although the direction of gravity can not be changed, for it is always vertical, it will amount to the same thing, if the body be simply rotated through any angle. The forces of the system remain of the same magnitude and act at the same points within the body, but the direction of the system has changed relative to any line in the body. If follows, therefore, that the centre of gravity of a body is *fixed relative to the body*.

Position of C. G.

The centre of gravity of a body is *not necessarily in the body itself*; it may be at a point outside the material of the body. Thus the centre of gravity of a circular ring of wire is at the centre of the ring, that of an empty beaker is in the air enclosed by it.

It is not necessary again that a body should be rigid in order that it may have a centre of gravity. Thus we speak of the centre of gravity of a fluid mass, or the centre of gravity of a system of bodies not materially connected in any way.

It is only sufficient for the centre of gravity of a body to be a definite fixed point relative to the body as long as its *size* and *shape* remain unaltered. If the body is made up of movable parts, the centre of gravity is fixed for any given configuration of the body, but changes its position with change of configuration. For example, a straight piece of uniform wire has its centre of gravity at its middle point, and its weight will act at that point as long as it remains straight. If the wire be bent, it will no longer have the same centre of gravity. In a draw telescope the position of the centre of gravity varies as the tubes are more or less drawn out. Again, if a man raises his arm, his C. G. is displaced.

The centre of gravity of a body may, therefore, be defined as the point, fixed relative to the body, through which the resultant of the weights of the particles which build up the body, passes for all positions of the body, so long as its size and shape remain constant.

Def. of C. G.

For all practical purposes the centre of gravity is the point at which the whole weight of the body may be supposed to act.

The centre of gravity is often abbreviated to **C. G.**

***60. Centre of Mass.**—If instead of considering the weight of a body we look to its mass only, and suppose the body to be acted upon by a system of parallel forces, such that the forces to which the individual particles of the body are subjected, are all parallel in direction and proportional to the masses of the particles in magnitude. The centre of the system of parallel forces, through which the resultant of the forces may be supposed to act, is the **Mass-centre**. It may be said to be

the point about which the mass of a body is evenly distributed. Every body, since it has a mass, has always a Centre of Mass (C. M·)

We observe that the centre of gravity is a particular case only of the mass-centre, in which the forces are the vertical forces due to gravity and the *C. G.* of a body does necessarily coincide with its *C. M.*

C. G. and C.M.

In Mechanics, when a body is moving in any way, in general, the velocity of its centre of mass is taken for the velocity of the body.

61. C. G. of Symmetrical Bodies.—In finding by calculation the cenre of gravity of different bodies the theory of finding the resulant of a number of parallel forces acting at known points is to be applied in methods adopted according to the forms of the bodies. From considerations of symmetry, however, we can at once note the following results. It is assumed that the bodies herein considered are of uniform density throughout.

The centre of gravity of

(1) a straight piece of a uniform wire, stick, rod or beam etc., is the middle point of its axis ;

(2) a uniform circular lamina or a sphere is its geometrical centre ;

(3) a uniform circular ring is the centre of the circle ;

(4) a uniform parallelogram lamina or a rectangular parallelopiped is at the intersection of its diagonals ;

(5) a uniform triangular lamina* is at the intersection of its medians.

62. To find by experiment the C. G. of a Lamina—The centre of gravity of a lamina may also be found by experiment. It is shown in art. 63 that when a body is suspended freely from a point and is in equilibrium, its C. G. is in the vertical line passing through the point of suspension.

* A *lamina* is a sheet of material of small thickness, such as a sheet of paper, a thin sheet of metal etc. An *uniform* lamina is of the same thickness and is formed of the same substance throughout.

Expt. 19. Suspend the body by a string attached to one corner of the body. Trace on it the vertical line through the point of suspension by means of a plumb-line.

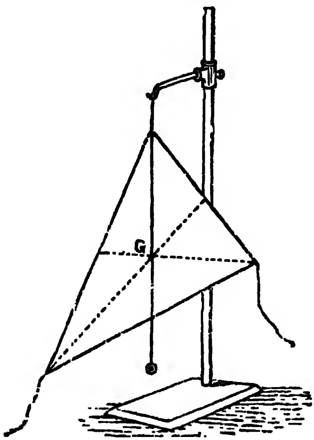


FIG. 53.
Determination of C. G. by suspension.

Hang the lamina from another point, and draw the vertical line through the point of suspension as before; G, the point of intersection of the lines will be the centre of gravity required. To verify this, suspend the lamina from a third point, the vertical through which will also be found to pass through G.

The C. G. of a lamina as that of a card-board or a sheet of metal plate may also be found by balancing the body in two different positions on a horizontal edge. When the body is balanced, its C. G. must be supported by the latter and hence must be vertically over some point of the edge.

Expt. 20. Balance a card-board against the edge of a table (fig. 54). Holding the card in this position draw by a pencil

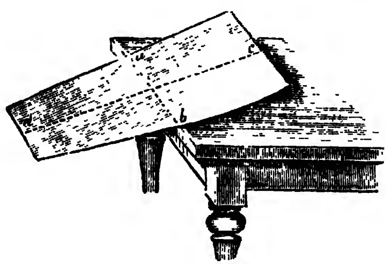


FIG. 54.
Determination of C. G. by Balancing.

a line on the under surface of the card, using the edge as

a ruler. Turn the card in some other position and repeat the process. The point of intersection of the two lines thus marked indicates the position of the C. G. wanted.

Support the body on a pin-head placed at the C. G. thus found. The plate thus supported ought to be in equilibrium.

63. Equilibrium of Heavy Bodies—A body at rest under the action of forces which balance each other is said to be in **equilibrium**. As the weight of a body may be supposed to act vertically downwards through its centre of gravity, the condition of equilibrium in all cases is that this force must be counter-balanced by the resultant of the reactions at the points of supports to the body acting vertically upwards and passing through the C. G. of the body. We notice the following cases :—

When a body is supported at one point, for example when it is suspended by a thread attached at one point (fig. 55, A), or when it is balanced on a pivot point (fig. 55, C), or when it rests on a plane touching it at one point (fig. 55, D), equilibrium is only possible when its C. G. either coincides with this point or is exactly above or below it in the same vertical line. For when a body is supported in this way the only forces acting on it are its own weight W acting vertically through G, its C, G. and the force sup-

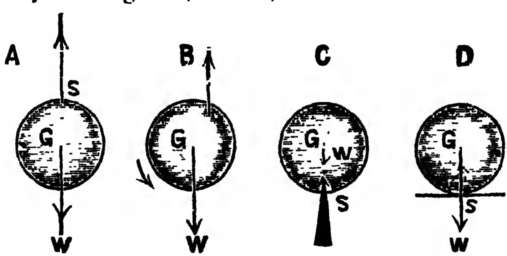


FIG. 55.
A body supported at one point.

porting it, and if these two forces are in equilibrium they must be equal and opposite and in the

same straight line. Thus in a plumb-line (fig. 55, A) the C. G. is vertically below the point of support. In any other position of the body, W acting at G, would cause rotation about S until G and S are brought along the same vertical line (fig. 55, B). In the case where a body is balanced on a point (fig. 55, C), the C. G. is vertically above the point of support. As an example of this latter we may take the case of a stick balanced on the finger.

Expt. 21. Try to balance a long stick on a finger-end. Note that unless care is always taken to keep the point of support vertically below the centre of gravity by the quick adjustment of the hand, the stick will fall. Also note that it is easier to balance a long stick than a short one. for the C. G. of the former in falling through greater height allows more time to adjust the point of support.

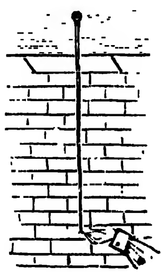


Fig. 56.
Balancing a stick

If the body is supported on two points A and B on a table, the reactions of the table at A and B are both upward vertical forces, the resultant of which must pass through some point between A and B in the line AB. Therefore the condition of equilibrium is that the vertical through the C. G. must fall between the two points of support on

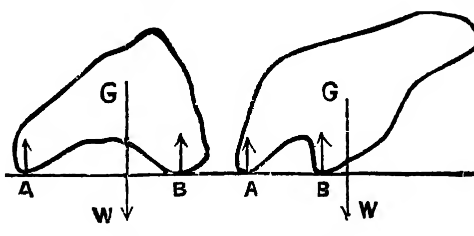


Fig. 57.

the line joining them. If the C. G. is in a position as shown in fig. 57. (ii). no equilibrium is possible,

if it have a position as in fig. 57 (i), it can remain in equilibrium. A man walking on *stilts* is an example of this case of equilibrium.

Lastly, consider the case of a body that rests on a plane surface on three points, for example a three-legged table ; or points more than three, for example a glass tumbler resting on a table. Imagine a fine thread drawn *tightly* round the body so as to include all the points of contact with the supporting surface ; the area thus enclosed is called the **base** of the body. Now the reaction of the table at the various points of support all act vertically upwards, their resultant therefore acts vertically upwards at some point within the area of the base and this, since it balances the

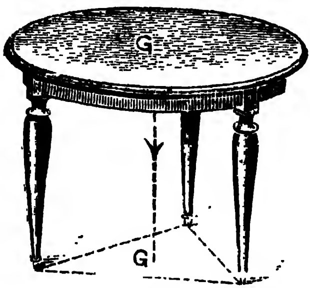


Fig. 58.
Equilibrium of a table.

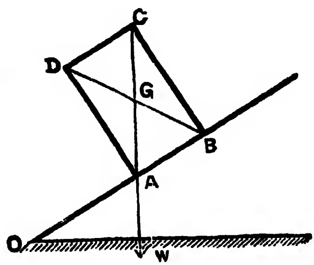


Fig. 59.
A body about to topple.

weight, must pass through the C G. Thus equilibrium is possible only when the vertical through the C. G. falls within the area of the base (fig. 58).

This condition is the same whether a body rests on a horizontal or on an inclined plane.

The truth of the above fact can be tested by some simple experiments :—

Expt. 22. Take a body of rectangular cross-section, say a brick. Place it on a *rough* plank and tilt the plank about an edge O. The body will not overturn so long as the vertical line through its C. G. falls within the base on which it rests (Fig. 59).

Now the C. G. of the body is evidently in the plane of the diagonal AC. Hence the brick will topple over just after the diagonal AC has passed through the vertical position.

Many illustrations on this point can be cited. If a cart is loaded with a very high load, so that the C. G. of the cart and load together is high above the ground, it may

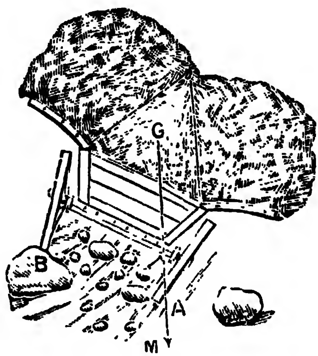


Fig. 60.
A loaded cart about to overturn.

be overturned by a small tilt caused by one wheel passing over a stone or a bank of earth, when the vertical through the C. G. falls outside the wheel base (fig. 60). For the same reason a boat is liable to upset easily, when the persons seated in it stand up, or when it is loaded to a great height.

In the ordinary upright position of a man the C. G. of his body is at about the middle of the lower half of the pelvis. But the C. G. is displaced when he carries a load. In order to retain stability he must modify his attitude so as to bring back his C. G. over a point between his two feet Thus a porter carrying a heavy trunk in one hand has to lean his body on the opposite side and often extends the other arm at full length. A man with a load on his back is obliged to lean forward. A person with an

overgrowth in his abdominal front (*pendulus belli*) has to throw back his head and shoulders. In rope-dancing the performer holds in his hand a long pole or an open umbrella to help him in maintaining the combined C. G. vertically above the rope.

Expt. 23. Stand sideways against a wall with a foot and the head both touching the wall ; and try to stand on one leg. It will seem to be impossible, because the wall will not allow the C. G. of the body to be brought over the foot next the wall.

Expt. 24. Sit on a chair and try to rise with the legs and the upper body kept in an upright position. It will be found to be impossible, unless the legs are thrown a little backwards and the body forward.

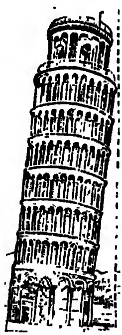


FIG. 61.
The Leaning Tower of Pisa.

If follows that the wider the base on which a body rests, the greater is its stability, for then even with a considerable inclination the vertical through its C. G. still falls within its base. The well-known *leaning tower of Pisa*, (fig. 61) from which were performed some of Galileo's famous experiments on falling bodies, is an illustration to the point. It is so much out of the vertical that it seems ready to fall at any moment ; yet it has remained in its present position for centuries.

64. States of Equilibrium.—Although we have seen that a body is in equilibrium when the resultant pressure of the supporting surface acts in the same vertical line passing through the C. G. of the body, it is yet possible to distinguish between the states of equilibrium. Equilibrium may be of three kinds, *Stable*, *Unstable* and *Neutral*.

A body is said to be in **stable** equilibrium, when it tends to its original equilibrium position after being *slightly* displaced. Thus, for example, the

plummmet in fig. 55 (A) is in stable equilibrium, for if pulled aside and then released, it will at first swing to-and-fro, but will come at last to rest in the same position.

A body is said to be in **unstable** equilibrium, when a body at rest, after receiving a small displacement, tends to move further away from its equilibrium position. Thus the stick balanced on the finger end (fig. 56) is in the state of unstable equilibrium.

Lastly, a body is said to be in **neutral** equilibrium, when after it is displaced, it rests in its new position. A uniform sphere (fig. 55, D) or a cylinder resting on flat surface are in the state of neutral equilibrium.

A cone or an ordinary glass funnel affords a good illustration of all the three kinds of equilibrium. When it rests on its base, it is in stable equilibrium (fig. 62 A. A′) ; when balanced on its apex, it is in unstable equilibrium (fig. 62, B, B′) and while resting on its side, it is in the state of neutral equilibrium (fig. 62 C, C′),

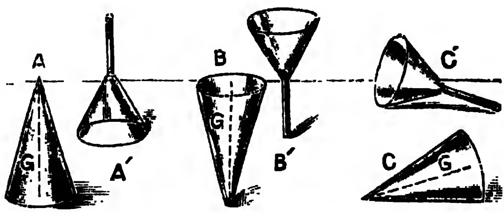


FIG. 62.
The three states of equilibrium.

The three states of equilibrium of a body are, in fact, determined by the position of the centre of gravity of the body. In all cases of stable equilibrium the C. G. is as low as possible. The slightest

displacement of the body elevates its centre of gravity. Hence the body returns to its original position as soon as it is permitted to do so. When the equilibrium is unstable, the centre of gravity is as high as possible. Any slight displacement of the body will help its C. G. in coming down to a lower height. From the above it is seen that the C. G. of a body *tends to occupy the lowest possible position.* In the neutral equilibrium of a body, its centre of gravity is neither raised nor lowered by any displacement given to the body.

States of Equilibrium and position of the C. G.

When a body can rest on a plane on different bases, for example a book, the limits of stability widen for a position which allows the centre of gravity to be lowered. Thus a copy of the Encyclopædia Brittanica has a greater stability when it lies flat on a table then when it stands on an edge. On the other hand, the stability of a body ordinarily in an unstable equilibrium may be increased by the addition of weights, so that the C. G. is brought under the point of support (fig 63).

Expt. 25. Fix two knives into a common cork on opposite sides (fig. 63). The cork will now very easily balance on the

Fig. 63.

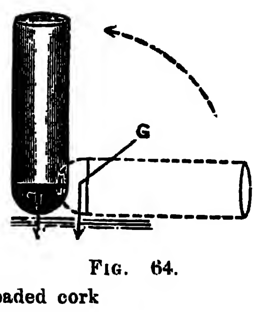


Fig. 64.

The C. G. of a loaded cork

point of a finger or a pencil, and will not fall even when the point of support is shifted to one side of the base of the cork.

Some apparently paradoxical but illustrative and interesting experiments may be arranged on the point :—

Expt. 26. Fix a cylinder of cork to a solid hemisphere of lead (fig. 64) by means of sealing wax. As lead is much heavier than cork, the C. G. of the whole is below the centre of the hemisphere. It will stand upright though it looks top-heavy, Tilt it to one side to make it horizontal. The point G, its C. G. is thereby raised ; the weight of the body acting there makes it spring back to its original position as soon at the body is free.

Fig. 65 represents a similar toy called the *Tumbler* which consists of a light figure attached to a hemisphere of lead. When the figure is upright, its C. G. occupies the lowest position.

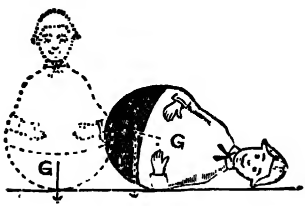


FIG. 5.

The tumbler.

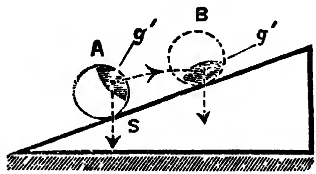


FIG. 66.

A loaded disc on a tilted plane.

Fig. 66 affords another illustration. It is a disc of wood with a small mass M of lead inserted within it near the edge. The point G is the combined centre of gravity.

Expt 27. Place the disc on a plane slightly inclined in the position A when the vertical through G is a little above the point of contact, S. The disc will ascend the plane, because thereby the point G comes to occupy a really lower position with respect to the ground. After gliding up the plane to a certain extent, it stops and then moves down and settles to a state when the vertical through G passes through the point of contact.

Exercise—VI.

1. Define Centre of Gravity. How can you find practically the C. G. of an irregular plate ?

2. State in general way when a body will stand or fall. Cite practical illustrations.

3. Define the three states of equilibrium. How does the position of the C. G. affect the equilibrium of the body ?

4. Explain the cause of the attitude of a fat man walking, of a man climbing up a hill, a person carrying a load on his back or in his hand, and a man rising from a stool.

5. A circular table weighs 20 lbs. and rests on three legs in its circumference forming an equilateral triangle. Find the least pressure that must be applied at its edge to overturn it.

6. A telescope consists of three tubes each 10 in. in length sliding within one another, and their weights are 8, 7, 6, ozs. Find the position of the centre of gravity when the tubes are drawn out to their full lengths. —*Lond. Matric.*

7. A cylinder, whose base is a circle 1 ft. in diameter, and whose height is 3 ft., rests on a horizontal plane with its axis vertical. Find how high one edge of the base can be raised without causing the cylinder to turn over.

8. Weights of 1 lb., 2 lbs, 3 lbs, and 4 lbs., are suspended from an uniform lever 5 feet long at distances of 1 foot, 2 feet, 3 feet and 4 feet respectively from one end If the mass of the levers is 4 lbs. find the position of the point about which it will balance.

9. A heavy beam consists of two portions, whose lengths are as 3 to 5, and whose weights are as 3 to 1 ; find the position of the centre of gravity.

10. A uniform plate of metal 10 inches square, has a hole of area 3 square inches cut out of it, the centre of the whole being $2\frac{1}{3}$ inches from the centre of the plate ; find the position of the centre of gravity of the remainder of the plate.

CHAPTER VII

FRICTION

65. Resistance to Motion.—The forces which tend to oppose or destroy motion are, in general, called *Resistances.* Thus a man dragging a heavy weight along the ground has to make a muscular effort to overcome the resistance of the ground opposing motion : a body falling in air or a cyclist riding against the wind has to meet with the resistance of the air ; a ship in motion has to cut the way through water ; in a machine, a part of the work done by it is always spent in overcoming the frictional forces between the different parts in the machinery. *Friction*, in the widest sense of the term may be used to mean any resistance to motion, but it is ordinarily used in a limited sense that *it is the resistance which a moving solid meets with on the surface of the another solid which supports it.*

Friction

In the cases of bodies moving through a fluid medium, the moving body has to set in motion those parts of the medium with which it is in contact. The reistance encountered by the body is directed against its front side and increases with the velocity and the extent of exposed surface of the body. It also increases with the density of the medium. The resistance of air serves to diminish the velocity of a rain-drop or a hail-stone which, falling from a height of mile or so, would otherwise have attained the speed of a musket shot. Use is made, on the otherhand, of this resistance of air in a descent by parachute and in regulating wind-vanes for diminishing the velocity of falling bodies.

66. Friction.—If two bodies be in contact with each other and a force be applied tending to make one body slide on the other, an opposing force is set up in the plane of contact of the surfaces in a direction tending to prevent the motion. This force is known as the *force of friction* between the surfaces in contact.

The friction is due to the *roughness* of the surfaces in contact. If these surfaces be perfectly smooth, there would be no opposing force of friction. Practically, however, the surfaces of bodies are never perfectly smooth. The minute irregularities of one surface engage with the small inequalities of the other and thus always cause some force to act between the surfaces in contact, being directed so as to prevent any displacement of one surface relative to the other in the plane of contact. In other words, the resistance due to friction acts in a direction parallel to the surface.

Let a rectangular body B (fig. 67) be placed with one of its plane faces resting on the plane horizontal surface of a table A. So long as B is at *rest*, the upwards pressure R of the table balances the weight W of the block: these forces are both vertical and there is no component in the direction of the surface, no friction being called into play. If a small force P be now applied to B parallel to the surface, a resistance F is felt which acting in the plane of contact prevents the block from sliding over the plate A. Sufficient friction is thus exerted just to stop the motion, F being exactly equal to P. As P is increased, the value of F also increases being always equal

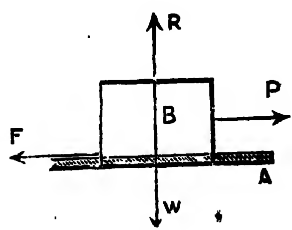


FIG. 67.
Friction on a horizontal plane.

to P. Friction is, therefore, *a self-adjusting force*: no more friction, however, is called into play than what is just sufficient to prevent motion.

But as P is increased indefinitely, the amount of friction F exerted at the plane of contact connot evidently be unlimited. The force F soon reaches a certain maximum limit depending on the nature of the surfaces in contact and the pressure exerted between them.

Limiting Friction.

The maximum limit to the value of the force of friction exerted when one body is just on the point of sliding upon another body, is called the **Limiting Friction.** When P is increased beyond this limit, motion of the block ensues.

Expt. 28. Clamp a large piece of wood on a table so as to be horizontal. It carries a light pulley attached to its one end. Place a rectangular block of wood to act as a sliding piece. The surfaces of the block and the plank of wood in contact should

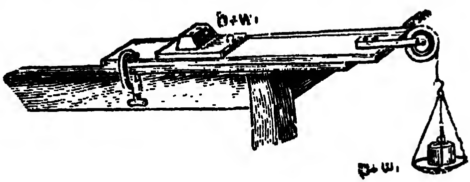


Fig. 68.

Friction Apparatus.

be made smooth and even by rubbing with sand-paper The block is attached to a string which passing over the pulley is connected to a pan. The part of the string above the table should be horizontal.

Begin the experiment by placing small weights on the pan and then go on adding until the block just begins to slip on the board. Near about the slipping point the board should be gently tapped to get a fairly close result. Note the total weight on the pan ; this together with the weight of the pan itself measures the maximum friction exerted between the board and the wood.

• Repeat the experiments. It will be seen that the value obtained for the limiting friction is fairly constant.

It is to be understood, however, that the limiting value of the friction found in the last experiments is constant in the particular case considered. Any change in the conditions in the above experiments will cause the limiting friction to change in magnitude.

Expt. 29. Determine the maximum friction *(a)* when a wooden block is placed on the wooden plank having tibres parallel and then at right angles to each other; (these will be different); *(b)* when the former is placed on a glass surface.

The result of this experiment shows that the maximum friction depends on the nature of the surfaces in contact.

67. The Laws of Limiting Friction.—The relation between the limiting friction between two surfaces in contact and the area of contact, as well as that between the limiting friction and the pressure exerted normally between the surfaces, are to be determined by experimental facts.

Expt. 30. Set up the apparatus as in Expt. 28. Determine the maximum friction between the block and the plank when the block rests on the plane successively on each of its three sides of different areas. Thus the areas of the surfaces in contact have been altered but the normal pressure between them, which equals the weight of the block, remains evidently the same.

It will be found that so long as the normal pressure between the surfaces is not changed, the limiting friction remains practically the same, and is independent of the area of surfaces of contact.

Expt 31. Determine the weight of the sliding piece A and let it be b. Find also p, the weight of pan. Set up the apparatus as in Expt. 28 and then determine the maximum friction using the unloaded block. The value of F is given by $p+w$, where w is the weight placed on the pan just sufficient to cause motion of the block.

Next place different loads on the block and determine each time the maximum friction. Tabulate your results.

The above expriment is performed without altering the area and the nature of the surfaces of contact. It

will be found that when the surfaces in contact remain the same, the ratio of the limiting value of the friction to the normal pressure between the surfaces, *i. e.*, F/R is constant.

We are then led to the following two laws of limiting friction, which though probably not rigorouly true, express fairly well the result of experiments :—

(1) The ratio of the limiting friction to the normal force between any two given surfaces is constant.

(2) This constant ratio depends on the material of the surfaces in contact and the state of their polish, but not on their area or shape.

It will be noticed in the performance of all the above experiments that after the sliding motion once takes place, the weight on the pan is sufficient to keep the block in motion and gives it a small acceleration. It follows from this that a slightly greater force is required to start a body moving on a rough surface against friction than to maintain it in motion when once started : in other words, the *statical friction* between two surfaces is slightly greater than the *dynamical friction* between the same two surfaces.

68. Coefficient of Friction—The constant ratio of the limiting friction to the normal pressure for any two specified surfaces is called the **co-efficient of friction**, and is generally denoted by μ. If F be the limiting friction and R the normal force, we have

$$F/R = \mu \qquad \text{whence } F = \mu R$$

The value of μ is obtained from the experimental determination of F as in Expt. 28. The determination may also be made by placing the body on an ordinarily smooth horizontal surface which is to be gradually tilted until the body just begins to slide down. When this is the case, the friction which acts there upwards to oppose the motion of the

body *down* the plane has reached its maximum value and just balances the component of the weight down the board. Let α be the inclination of the plane to the horizon. Let W be the weight of the body and R, the normal component force (fig. 69). It follows from geometry that the angle which the normal to the plane makes with the vertical is equal to the angle of the plane.

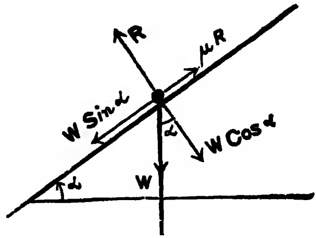


FIG. 69.

Friction on a rough inclined plane.

Resolving W in directions perpendicular and parallel to the plane, we have

$$F = W \sin \alpha$$

and $$R = W \cos \alpha$$

whence $$F/R = \frac{W \sin \alpha}{W \cos \alpha} = \tan \alpha$$

But $F/R = \mu$, the coefficient of friction

$$\therefore \quad \mu = \tan \alpha$$

Tan α again is obtained from the relation

$$\tan \alpha = \frac{\text{height}}{\text{base}} \text{ of the plane.}$$

Thus the co-efficient of friction is found by reading the height of the plane and dividing it by the base

The approximate values of the coefficient of friction in a few common cases are given below :—

Surfaces in contact.

Wood on wood, fibres parallel	.0·5
,, ,, fibres a right angles	...0·33
Metal on wood	0·18
Metal on metal	0·6
Leather on metal, dry	.0·56
,, ,, oily	..0·15

69. Rolling and Sliding Friction.—Friction is of two kinds, *sliding* and *rolling*. When a body slides over another, for example when a heavy box is dragged along a floor, when the two hands are rubbed together, or when an axle of a wheel rotates, sliding continuously upon the same points of its bearings, the case is one of sliding friction. When a body, on the other hand, rolls over another, as in the case of an ordinary wheel on a road, the case is of one of rolling friction. As rolling friction is considerably less than the sliding friction, there is a great saving of power when the latter is converted into the former.

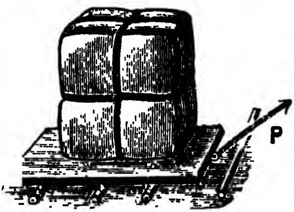


FIG. 70.
Roller Bearing.

Hence the use of castors on heavy pieces of furniture such as pianos, tables etc. Heavy weights like large blocks of stones are dragged along by supporting them on rollers (fig. 70). In the ordinary wheels, the sliding friction is not, however, entirely removed, for the wheel slides continuously at some point A upon the axle (Fig 72). In the *ball-bearing* arrangement, as in bicycles, the sliding friction is completely replaced by rolling friction where a number of hard steel balls are loosely confined in a metal case round the axle ; the hub of the wheel rolls on them (fig 71).

To diminish friction various other methods are adopted. It is diminished by polishing the surfaces in contact. Lubricators such as oil, graphite, tallow are in frequent use to diminish the frictional resistance in machines. In the motion of sledges over a ground of ice, the ice melts under pressure ; its surface acts like a lubricated polished surface and the friction is much reduced. As a rule, greasy substances

which are not absorbed by a body diminish friction but increase it if they are absorbed ; thus moisture and tallow increase the friction of wooden surfaces but diminish that between metal surfaces. Again trains and tramcars are made to run on iron rails. Rims of

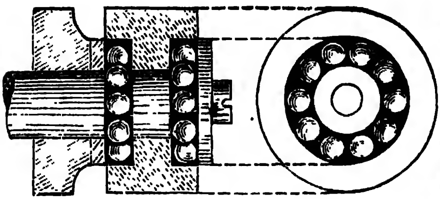


FIG. 71.
Ball-Bearing

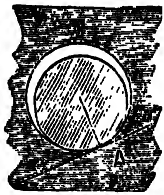


FIG. 72.
Common Bearing.

wheels in various carriages are provided with india-rubber tyres. Roads are covered with tar-macadam or materials like it which, when consolidated, form a firm and even surface. Agate centres in compasses, jewel centres in watches and agate knife-edges in balances are supported on steel pivots.

Again, friction plays a necessary part in the mechanical problems of our everyday life ; without friction on the ground neither men nor animals would be able to walk ; no transmission of motion by belting, rope etc in a machine, or by a locomotive engine would be possible. Without friction nails, screws would not remain in bodies ; all knots, textils fibres, would fall to pieces. It is indeed sometimes desirable to increase friction *e. g*, sawdust is strewn on ice, sands are thrown on railway lines after rain, a violin bow is rubbed with resin to increase its friction with the strings etc., etc.

Frictional forces differ in one respect from other forces in nature such as gravitative, magnetic, electric etc. When a body is raised up against the action of gravity, work is done upon it which is stored up in it as its potential energy which reappears in another form when the body is allowed to come down from its,

elevated position, and the body can in its turn do work upon another body But *the energy spent in drawing a body against friction is not so recoverable.* If motion of a body from A to B is opposed by friction, its motion from B to A is also opposed and not helped by friction. The energy is converted into heat and is lost to us for further use.

Exercise.—VII.

1. What do you mean by *Friction?* Define co-efficient of friction.

2. The co-efficient of friction between a block and a table is 0·3. What force will be required to keep the block weighing 400 gms. in unform motion ?

3. A heavy body is just on the point of sliding on a rough plane that rises 3 in a length of 5 ; find the co-efficient of friction.

4. A body, of weight 6 lbs. rests in limiting equilibrium on a rough plane whose slope is 30. The plane is next raised to a slope of 60 ; find the force along the plane required to support the body.

5 Give instances of cases in which it is desired to have the friction increased, and others in which to have it diminished.

6 Describe a method of determining the co-efficient of friction between teak and iron.

An inclined plane is adjusted so that a flat-bottomed box placed on it just steadily moves down. What difference will you notice if a load of a kilogramme is placed on the box ? Give reasons for your answer. [*Pat. U.—1919*

CHAPTER VIII.

MACHINES.

70. Machines.—A *machine* is a contrivance or an instrument by means of which a force, applied at a point and in a given direction, is able to exert, at some other point, a force differing in direction and intensity, which is applied to overcome some other resisting force.

Utility of a machine

Thus by means of a pulley and a rope, a weight such as a bucket of mortar is raised to a height, one end of the rope being pulled by a man standing on the ground. The force applied is the same in intensity as the weight overcome, but is applied in a more convenient direction. Again, by applying a small force at one point, there may be a gain of force due to a machine, *i. e.*, a larger force is available at some other point, *e. g.*, a nut-cracker, a cork-presser. Thirdly, a machine may take a larger force to produce a smaller force thus causing a loss in force, but it will cause at the same time a *gain in time*, *i. e.*, rapidity of action or motion *e. g.*, a bicycle.

A machine is, as is thus seen, always used in practice to overcome some resistance. The force impressed on the machine is called the **Effort** or **Power**, and the resistance to be overcome is ordinarily called the **Weight** or simply the **Resistance**. The effort is generally denoted by P, and the resistance by either Q or M.

The term '*Power*' is not a happy one to use in this sense, for it is now definitely used to mean *the rate of working*; hence

the word *effort* is preferred to it. Again, it is better to use the term *Resistance* instead of *weight* as machines are often used in overcoming resistances other than those due to gravity.

71. Work done by a Machine.—It must be noticed at the outset that whether the force exerted by a machine is greater or not than the force impressed on it, the machine is unable to supply a greater quantity of energy than what is put into it; in other words, no more work is done *by* it than is done *on* it. As a matter of fact, there is bound to be present in every machine some amount of friction existing between its different parts Hence a part of the energy supplied to a machine is lost in overcoming this internal resisance within it, and the rest is the effective work done by it. Thus the larger the friction, the less is the energy utilized by the machine.

The ratio, which expresses what part of the total energy supplied to a machine is utilized by it, is called the **Efficiency** of the machine. Thus

$$\text{Efficiency} = \frac{\text{Energy utilized}}{\text{Total energy supplied}}$$

The efficiency of a machine is always less than unity and is often multiplied by 100 and stated as a percentage. If we suppose a machine to be frictionless, or that the friction therein is negligible, the efficiency equals unity.

Thus it follows from the fundamental principle known as the *Principle of Work* or Conservation of Energy that

'Whatever be the machine used, provided that there be no friction within it and that the weight of the machine be neglected, *the work done by the effort is always equivalent to the work done against the resistance.'*

Now work is measured by the product of two factors, a *Force* and the *Displacement* of its point of application. It follows that if the machine be perfectly smooth

throughout, and if P and W be the two forces which balance in the machine, then

$$P \times \text{distance through which } P \text{ moves}$$
$$= W \times \text{distance through which } W \text{ moves.}$$

Thus a small impressed force multiplied by a large displacement may overcome a larger force multiplied by a small displacement : a machine enables this to be done.

Further, in such a case when P is smaller than W, the point of application of P will have to move through a longer distance than that through which the point of application of W moves. This is popularly expressed as,—*what is gained in power, is lost in speed.*

72. Mechanical Advantage—It is the ratio of the resistance overcome to the effort supplied to a machine. Thus

$$\text{Mech. Advantage} = \frac{\text{Resistance}}{\text{Effort}} = \frac{W}{P}$$

From the equation of work in art. 71, this

$$= \frac{\text{Distance through which } P \text{ moves}}{\text{Distance through which } W \text{ moves}}$$

Machines are generally so constructed that their mechanical advantage is greater than unity. But though there is mechanical advantage, no work is ever gained by the use of a machine

Machines are also employed for exerting a larger force to overcome a smaller force. With respect to the force applied such machines work at a much disadvantage ; but with respect to the distance traversed, there is an advantage, rapidity of action being secured.

73. Simple Machines.—An ordinary machine, *e.g.*, a pump, a steam-engine etc, consists of a number of simple parts which may be classified for separate study. Each of these parts is spoken of as a *simple machine.*

Simple machines are also called **Mechanical Powers.**

The Simple Macnines may be classified as :—

(i) The Lever, including the Wheel and Axle ;

(ii) The Pulley ;

(iii) The Inclined Plane, including the Wedge : and

(iv) The Screw

74. The Lever.—The *lever* is a rigid bar, straight or bent, and is capable of turning about a fixed point of support. The fixed point is called the **Fulcrum**, and is denoted by F in the figures.

The perpendicular distances between the fulcrum and the lines of action of the Effort and the Resistance are called the *arms* of the lever. Thus in fig. 73 the arms are FL and FM.

The conditions of equilibrium in any case of lever are obtained from the Principle of Moments. The resultant of the forces P and Q, impressed at A and B respectively, must pass through F. Hence the moment of P about F must be equal to that of Q about F. Thus

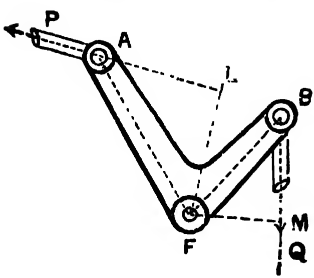


FIG. 73.
A Bent Lever.

$$P \times FL = Q \times FM.$$

The lever is most often a straight rod. We shall consider the cases when the lever is a straight one, and the effort P and the resistance Q are perpendicular to it. In theoretical calculations the thickness of the lever rod and its weight are neglected

Levers of this kind are usually divided into three classes according to the position of the fulcrum with respect to the points of application of the effort and resistance.

75. The Three Classes of Levers; Class I.—In this class the effort P and the resistance Q or W acting on opposite sides of the fulcrum F keep the lever in equilibrium (Fig. 74).

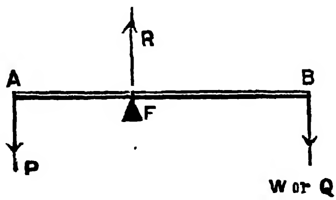


FIG. 74.
Lever, Class I.

Let R be the resultant of P and W, acting through F. As the lever presses *downward* on F with the force R, the reaction at F acts upwards with the same force R.

As a condition of equilibrium, we have

$$R = P + W \qquad \dots \text{(i)}$$

as P and W are like parallel forces.

And the moment of P about F = the moment of W about F

Or $P \times \text{P-arm} = W \times \text{W-arm} \qquad \dots \text{(ii)}$

i.e. $P \times FA = W \times FB$

The mechanical advantage is given by

$$\frac{W}{P} = \frac{FA}{FB} = \frac{a}{b}$$

where a and b are the lengths of the arms FA and FB.

In this lever as a may be greater than, equal to, or less than b according to the position of F along the rod, the mechanical advantage may also be greater than, equal to or less than unity.

The ordinary balance, in which the two arms are equal, is a special case of this class of lever.

Instances of the levers of this class ;—A crow-bar, as ordinarily used to raise a weight, having its fulcrum

at a point where it rests on a block near to the weight to be lifted (fig. 75). A poker, used to raise coal

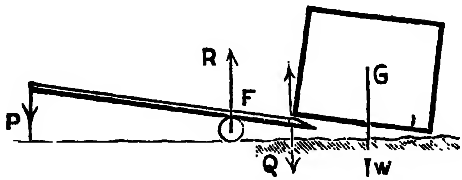


FIG. 75.

A crow-bar used in raising a weight.

in a grating ; a claw-hammer, when used to extract nails ; a spade, in digging the earth : a see-saw ; the handle of an ordinary pump ; an ordinanry balance : the foot, when it is raised and the toe tapped on the ground, the ankle-joint being the fulcrum

The oar of a rowing boat may be regarded as a lever of the first kind with the rowlock as the fulcrum, if the boat were kept at rest and the oar used to scoop the water backwards.

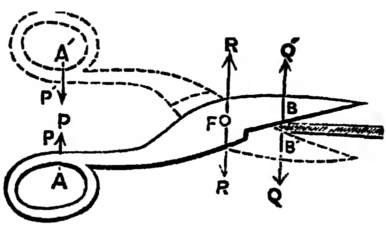


FIG. 76.--A pair of scissors.

A pair of scissors (fig. 76), a pair of crucible tongs are instances of Double Levers of the First Class.

The principle of lever is said to be discovered by ARCHIMEDES, the noted geometrician of antiquity. It is related that at the launching of a huge ship designed by him Archimedes displayed the power of a lever by using it for urging the ship off the stocks, and that in reply to king Hiero's expression

Archimedes on the lever. B.C. 250.

of wonder at the great force thus displayed, Archimedes uttered his famous boast "Give me but a place to stand on, and I will raise the world."

Class II.—In the second class of levers, the Effort and Resistance act on the same side of the fulcrum F, but in opposite directions, the effort acting at a greater distance from the fulcrum than the resistance (fig 77).

Here we have

$$R = W - P$$

and

$$P . AF = W . BF$$

Hence

$$\frac{W}{P} = \frac{FA}{FB} = \frac{a}{b}$$

And since a is greater than b, the mechanical advantage is always greater than unity.

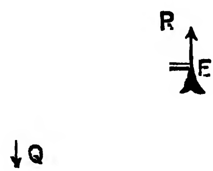


FIG. 77.
Lever, Class II.

Examples of this kind of lever :—a wheel-barrow, in which the fulcrum is at the axle of the wheel and the effort is applied at the handle :—a crow bar, when one end of it is in contact with the ground and effort is applied at the other end : an oar, when the boat moves forwards, the end of the oar in water being the fulcrum which is, of course, not absolutely fixed : a cork-squeezer.

The raising of the body upon the toes in standing on tip-toe, or in the first stage of making a step forwards is an instance of the lever of the second class in the human body. Here the fulcrum is the ground on which the toes rest : the power is applied by

the muscles of the calf to the heel: the resistance is the weight of the body borne by the ankle-joint.

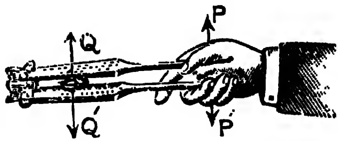


FIG. 78.
A pair of nut-crackers.

The consideration of this kind of lever explains why a finger caught near the hinge of a shutting door is so severely crushed.

A pair of nut-crackers (fig. 78) is a double lever of this class.

Class III:—The fulcrum in this case is at one end, and the effort P and the resistance W act on the same side of F as in class II, but the effort acts nearer the fulcrum than the weight (fig. 79).

Here we have $R = P - W$

and $P. AF = W \times BF.$

therefore $\frac{W}{P} = \frac{FA}{FB} = \frac{a}{b}$

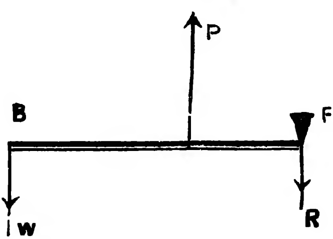


FIG. 79.
Lever,—Class III.

and since here a is less than b, the mechanical advantage is less than unity. A small weight to be raised

requires a larger effort but the point of application of weight is considerably displaced when that of the effort moves through a small distance.

The treadle of a lathe or a sewing machine is an example of this kind of lever. A very good example is seen in the bone of the fore-arm (fig. 80), where the fulcrum is the elbow-joint; the effort is applied by the contraction of the Biceps muscle, the lower end of which is attached to the fore-arm not far from the joint, and the weight is placed on the hand. Here rapidity of action is obtained at a loss of power. Cracking a nut by the teeth is another example of this kind.

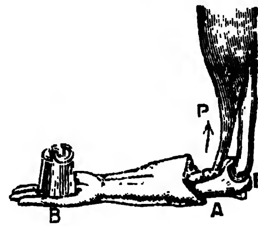


FIG. 80.
The fore-arm as a lever.

A pair of sugar-tongs, a pair of forceps in a weight-box are double levers of this class.

Practical verification of the relation between the effort and weight in a straight lever can be done in the same way as shown in Expt 14, Art. 48.

76. Wheel and Axle.—The wheel and axle is a modification of the lever. It consists of two cylinders having a common axis (fig 81), the larger of which is called the *wheel* and the smaller the *axle*, the axis common to both being horizontal The axis terminates in two pivots which can turn freely on fixed supports. Round the axle is coiled a rope, one end of which is fixed to the axle, while the other end supports the weight W. Round the wheel is coiled a second

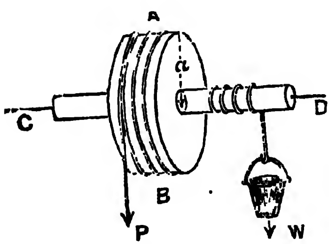


FIG. 81.
Wheel and Axle.

rope in the opposite direction to the first, having one end attached to the wheel and having the power P applied to the other end. Thus when P is lowered, the rope round the wheel unwinds, that round the axle coils up and raises the weight.

The condition of equilibrium is that the moments of the effect P and weight W about the axis must be equal and opposite.

i.e. $P.a = W.b$

where a and b are the radii of the wheel and axle respectively.

Hence the mechanical advantage is

$$\frac{W}{P} = \frac{a}{b} = \frac{\text{radius of wheel}}{\text{radius of axle}}$$

By making the wheel larger and the axle smaller, the mechanical advantage may be increased: but, in practice, this has a limit, for the axle cannot be made too thin and thereby too weak, nor the wheel too large and cumbrous.

The above result can also be obtained from the *Principle of Work*. Let the wheel and axle be rotated through one complete turn. Then a length of rope equal to $2\pi a$ uncoils from off the wheel and a length eqal to $2\pi b$ coils round the axle.

Hence work done by P

$= P \times 2\pi a$

and work done against W

$= W \times 2\pi b$

$\therefore \quad P \times 2\pi a = W \times 2\pi b$

whence $P.a = W.b$.

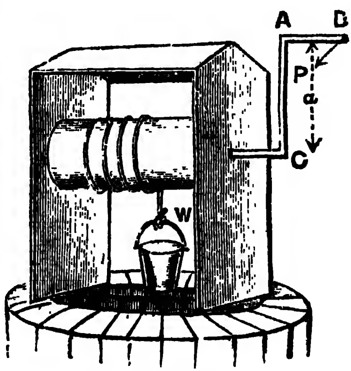


FIG. 82.—Windlass.

The windlass (fig. 82) and the capstan (fig 83) are modifications of the wheel and axle. In the *windlass* which is used for drawing water from a well, the wheel is replaced by a crank-handle. In the *Capstan* which

is used on board a ship for raising the anchor, the axis is verical One or more men may apply force by pushing a number of horizontal projecting arms (called hand-pikes). Here the moment of the pull on the rope is equivalent to the sum of the moments of forces exerted by the men.

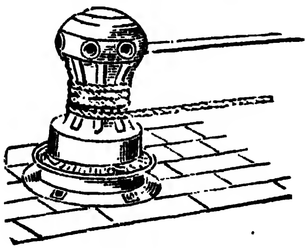


Fig. 83.
Capstan.

The method generally employed to obtain rotatory motion by means of a belting passing round two wheels, or a linked chain passing over two toothed wheels (as in a bicycle) is another application of the wheel and axle arrangement. If the circumference of the large driving wheel is double that of the smaller wheel, the latter will rotate twice as many times as the former in the same time.

A train of cog wheels (fig. 84) which is used in clocks, watches and speed-recorders is virtually a combination of

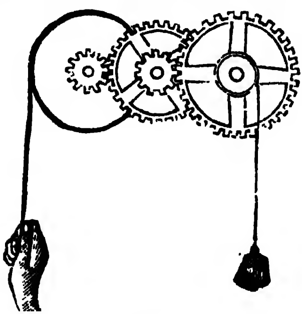


Fig. 84.
Cog-weels.

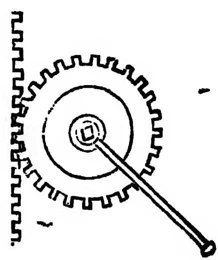


Fig. 85.
Rack and Pinion.

wheels and axles. Every wheel with its axle or pinion on the same axis is a wheel-and-axle arrangement. The

mechanical advantage of the whole system is the product of the mechanical advantage of each pair.

Fig. 85 represents a *Rack and Pinion* arrangement. It may be looked upon as a variety of the wheel and axle arrangement, in which the rack, a straighs bar fitted with teeth, is to be regarded as a portion of a wheel of an infinitely large diameter. When the pinion wheel is rotated on a fixed shaft, its motion is converted into a straight one in the rack. The piston of a double-barrelled air-pump, the focussing arrangement of a telescope or of a microscope etc., are worked by such a contrivance.

77. The Pulley.—The *pulley* is a small circular disc or wheel of wood or metal with a groove cut round its rim to receive a string or cord which passes over it. The pulley can revolve freely about an axle, passing through its centre perpendicular to its plane, the ends of the axle being supported in a frame-work, called the *Block*.

If the block be fixed as in fig. 86 the pulley is said to be *fixed*. When the block can ascend or descend as in fig 87, the pulley is said to be *movable*.

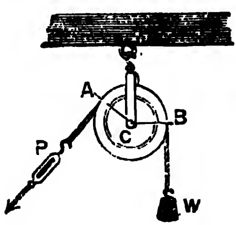


FIG. 86.
Fixed Pulley.

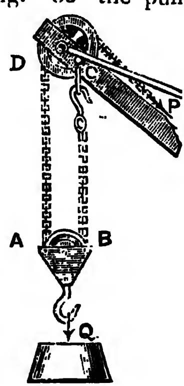


FIG. 87.
Movable Pulley

For elementary study we suppose the pulley to be smooth, so that the tension of the string passing round it is the same throughout. Further, the weights of the pulley

and the rope are often so small compared with the weights supported, that these may be regarded as negligible.

In the fixed pulley, the weight is attached to one end of a string passing over the groove, and the power is applied by pulling the other end. The fixed pulley is useful only in changing the direction of a force. Such a pulley is used for drawing curtains, hanging lamps, pulling punkhas, raising weights etc.

Taking moments about the centre of the wheel, we have

$$P. CA = W. CB$$

But $$CA = CB$$

Hence $$P = W$$

In a *Single Movable Pulley*, the weight is attached to a block, a string passing round the pulley is secured to a fixed support; the power is applied at the other end (fig. 87).

When the string are *parallel*, the tension along the two parallel strings supports the weight acting downwards. Moreover, if the puiley be assumed to be frictionless, the tension in any part of the string is P.

Hence $$2P = W$$

$$\therefore \text{ Mech. Advantage } = \frac{W}{P} = 2$$

i.e., a given power can in this case raise twice its weight.

Pulleys are often combined in various ways in order to secure higher mechanical advantages. The most common arrangement known as the **Block and Tackle** is employed on account of its superior portability (fig 88). In this, the pulleys are arranged in two blocks, one fixed and the other movable and attached to the weight. The string which is continuous is attached to one of the blocks and passes alternately, power being applied at the free end of the string.

The pulleys are sometimes arranged with a common axis; again, the various pulleys in a block are sometimes placed one below the other as in fig. 89.

In either case the tension of the string is equal to the power. Let n be the number of portions of string at *the lower block*.
Then the total upward force

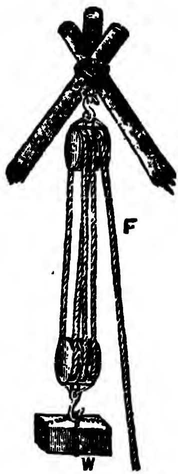


FIG. 88.
Block and Tackle

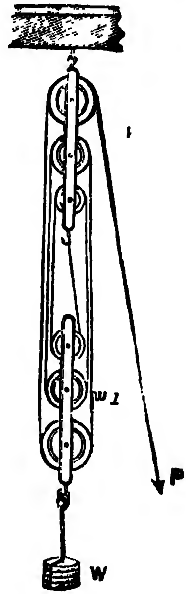


FIG. 89.
Second system of Pulleys.

at the lower block is nP. The downward force is W, the weight supported, together with the weight of the lower block (w).

$$\text{Hence} \qquad nP = W + w$$

78. *The Inclined Plane.—A plane, inclined to the horizontal plane at any angle α, is an *Inclined Plane*. By means of it a heavy body can be raised to a height by the application of a force less than the actual weight of the body, the friction on it being supposed to be negligibly small.

In fig. 90, ABC is an inclined plane with an inclination of α. AB is called the *length* (l), BC the *base* (b), and AC the *height* (h) of the plane.

Let W be the weight of the body, and P the force continuously acting on it in a direction parallel to the plane.

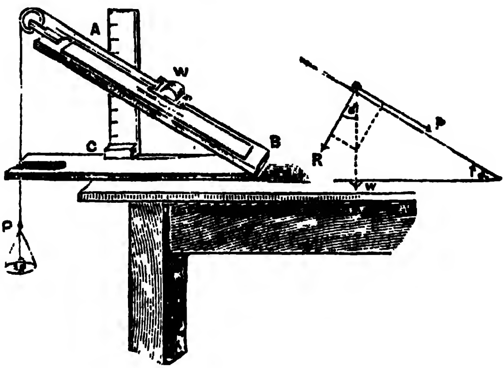


FIG. 90.
An Inclined Plane.

Now work done by P in drawing the body *up* the whole length of the plane $= P \times l$;

And work done by W against gravity $= W \times h$

By the *principal of work* $P \times b = W \times h$

$\therefore$ Mechanical advantage $= \dfrac{W}{P} = \dfrac{l}{h}$

The above can also be proved by the principle of *Resolution of Forces* :—

The component of W perpendicular to the plane

$= W \cos \alpha = R$, the normal reaction of the plane.

The component of W parallel to the plane, which only is effective in dragging the body down the inclined plane is

$= W \sin \alpha$. This must be balanced by P.

$$\therefore \quad P = W \sin \alpha$$

Mech. advantage $\dfrac{W}{P} = \dfrac{1}{\sin \alpha} =$

Expt 32. Verify the relation

$$\frac{P}{W}=\frac{h}{l}$$

when P acts parallel to an inclined plane*

If a railway engine has to pull a train up hill with an inclination of 1 in 50, it has to exert a force equal to only of the weight of the train in addition to what it will have to exert on a level railway merely to overcome the friction. The principle is applied in practice in loading and unloading heavy goods into or out of a wagon by means of two inclined beams connected by iron ties ; in following a zig-zag course in climbing a hill or going up a long stair-case etc. Again, the smaller the slope, the easier is the ascent ; this is also seen in the case of a common ladder or a staircase in a house.

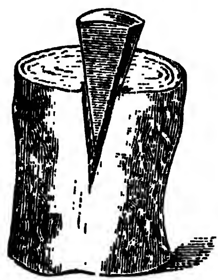


FIG. 91.
Wedge.

The Wedge—is a double inclined plane, movable instead of being fixed as in the case considered, made of iron or some hard material and used in splitting wood (fig. 91), lifting a weight such as raising large blocks in order to put rollers or chains under them. Knives, chisels, axes, choppers and many other cutting instruments are thin wedges with sharp edges.

79. The Screw.—Every one is familiar with a screw. It is essentially an inclined plance wound on a cylinder.

Expt. 33. Take a piece of paper ABC cut into the shape of a right-angled inclined plane ot small angle Colour the edge AB and wrap the paper round a cylinder, say a common

* See *Author's* Intermediate Practical Physics.

pencil (fig. 92), so that the base is at right angles to the axis of the cylinder. The edge AB will form a spiral curve on the cylinder and will trace a screw.

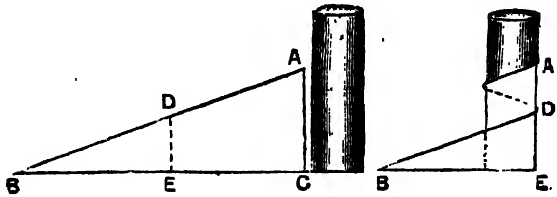


FIG. 92
The screw.

Thus a screw consists of a cylinder of metal whose surface carries a uniform projecting thread, or has a groove cut on it along a spiral curve, making a constant angle to lines parallel to the axis of the cylinder. The section of the thread of the screw may be of different shapes *e g.*, square, V-shaped etc.

The angle which the screw-thread makes with a plane at right angles to the axis is called the **Angle** of the screw. This is the angle ABC of the inclined plane which traces the screw.

The distance between two consecutive threads, measured parallel to the axis such as AD, is called the **Pitch** of the screw. If there be 15 threads for each inch of the axis of the cylinder, the pitch of the screw is said to be 1/15 inch, or 15 threads to an inch.

The screw usually works in a **nut** or **collar** C (fig. 93). *i.e.*, a fixed support in the inside of which a hole is bored having a groove to fit the thread or a thread to fit the groove of the screw as the case may be. When the screw turns in a fixed nut or collar, it moves forward or backward in the direction of its length. In each turn of the screw

the distance moved through is equal to that between two consecutive threads.

Let P be the effort acting at the end of an arm a in a direction at right angles to the axis, as in a vice. Let Q be the resistance overcome acting along the axis; and let b be the pitch of the screw.

Then if the screw is given one complete turn, the point of application of P will describe a circle of radius a. Hence its displacement $=2\pi a$. Also the screw moves through a distance b against the resistance Q.

It follows, from the *Principle of Work*, that

$$P \times 2\pi a = Q.\,b$$

$$\therefore \quad \text{mechanical advantage} = Q/P = 2\pi a/b$$

$$= \frac{\text{circumference of a circle of radius } a}{\text{Pitch of the screw}}$$

Thus by decreasing the pitch of a screw and making the arm at which P acts, longer, the mechanical advantage may be considerably increased.

The screw is extensively used in machines for fixing the different parts to one another. As a contrivance for exerting great pressure it has its practical application in a vice, printing-press, oil-press

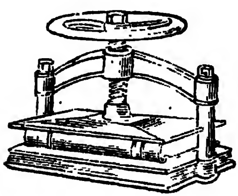


FIG. 93.
Screw-press.

FIG. 94.
Jack-screw.

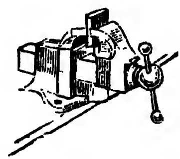


FIG. 95.
Vice.

etc. The jack-screw is used for lifting weights. Screws are used in the laboratory to produce small motion as in the levelling screws. Very small lengths are again measured by a micrometre screw, *e.g.*, a screw-gauge, a spherometer.

The *end-less* screw (fig. 96) is a combination of a screw with a wheel and axle. The screw-thread fits in a toothed wheel in such a way that when the screw rotates, the wheel moves forward one tooth for each turn. It is to be noted that here the screw does not advance. A rapid motion of the screw-shaft is converted into a slow motion of the wheel. The endless screw is employed in many instruments for registering speed.

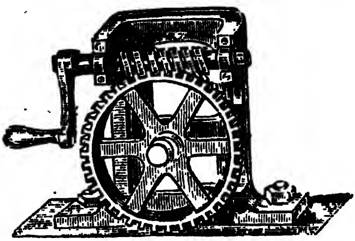


FIG. 96.
End-less screw

80. The Balance.—The balance is used for comparing masses of two bodies, or rather for determining the mass of a body in terms of a standard mass. This is, of course, done by comparing the weights of the bodies, as weights are proportional to their masses at one and the same place.

A description of the common balance has already been given in art. 11. It consists of a lever of the first kind with its fulcrum in the middle and placed a little above its centre of gravity. From the ends of the arms of the leaver two equal and similar scale-pans are suspended ; the mass to be weighed is placed in one of these and is balanced by placing suitable weights in the other. The arms of a balance ought, as we shall see later on, to be equal and similar, if the balance is to be accurate.

In a good balance, steel knife-edges are used to diminish friction, one at the fulcrum and two at the ends of the two arms. The fulcurm consists of a wedge-shaped piece of hard steel, whose fine edge is horizontal and perpendicular to the length of the

beam and rests on hard plates of steel or agate. The scale-pans are attached to plates of steel or agate, which rest on similar knife-edges, fixed at the extremeties of the beam with their edges turned upwards. A *needle* or a *pointer* is fixed to the beam near the fulcrum : the lower end of the pointer moves over a horizontal scale, such that when the beam is horizontal, the pointer is vertical and points to the zero graduation of the scale.

For the precautions to be taken in using a balance and for the method of weighing known as the Method of Oscillation *see* Practical Physics by the author.

81. The Requisites of a Good Balance.—A good balance should be so constructed that it is

(1) *true*, (2) *sensitive and* (3) *stable.*

(1) A balance is said to be **true**, if the beam be horizontal whenever bodies of equal weight are placed on the scale-pans.

The conditions required so that a balance may be true are that—

(i) *the centre of gravity of the beam must be vertically below the fulcrum when the beam is horizontal.* For, the condition of stable equilibrium requires that G, the C. G. and F, the fulcrum must be on the same vertical line. Then if G be above F, the beam would be unstable ; if G coincides with F, the beam would not oscillate ; but if G be below the fulcrum, the weight of the beam in its inclined position is continually tending to bring it back to the original horizontal position ; in other words, the balance oscillates with regularity.

(ii) *The two arms of the balance must be equal.*

In fig. 97, let

W' – weight of the beam acting through G, its centre of gravity,

S, S' – weights of the scale-pans respectively,

P, Q – weights of the masses on the scale pans,

and $FG = k$, $AP = a$ and $BC = b$.

Now suppose the pans are empty and the beam is horizontal. The only forces which have a movement about F are the weights

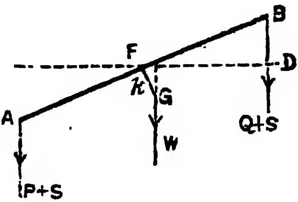


FIG. 97.

S and S' of the pans acting vertically through A and B respectively. Taking moments about F, we have

$$S.a = S'b.$$

Let two equal masses P and P be now placed on the two scale-pans: if the balance be true, the beam will still be horizontal. We have then

	$(P+S).\ a$	$=$	$(P+S').\ b$
Or	$P.a+S.a$	$=$	$P.b+S'.b$
But	$S.a$	$=$	$S'.b$
Therefore	$P.a$	$=$	$P.b$
Or	a	$=$	b
And since	$S.a$	$=$	$S'.b$
We have also	S	$=$	S'

Therefore the third condition of a true balance is that (iii) *the scale-pans must be of equal weight.*

Expt 34. Test a balance as to its truth in the following way :—Put sufficient weights on one scale-pan to balance the weight of a body placed on the other pan. Next interchange the body of the weights. If they still balance each other by keeping the beam horizontal, the balance must be true.

(2) A balance is said to be **sensitive** when the beam deviates appreciably from its horizontal position for a very small difference between the weights in the scale-pans. A good chemical balance will indicate a difference of weight down to a tenth of a milligramme.

For a given value of $P-W$, the greater the inclination θ of the beam to the horizon, the more sensitive is the balance ; also the less the difference of P and Q required to produce an inclination θ, the greater is the sensitiveness of the balance.

To find the conditions we assume that the balance is true. Let the centre of gravity of the beam be at G (fig. 97), a distance k below the fulcrum. When the beam is inclined, its weight W will have a moment equal to $W.k \sin \theta$ tending to restore it. In order that a balance may be sensitive, *i e.*, the inclination for a small difference in P and Q may be considerable, this moment of W about F must be small. This can be secured by making either k, or W, or both small.

Thus the conditions that a balance may be sensitive are that

(i) *the centre of gravity of the beam shall be very near the fulcrum ;*

(ii) *the beam should be light ;*

and (iii) *the arms should be long.*

The last condition is obtained from the fact that the inclination of the beam will also be great, if the moment of the difference of the weights in the pans is large. This moment, since the balance is true, is $(P-Q).a$. Thus the sensitiveness of a balance may be increased by increasing a, the length of the arms.

(3) A balance is said to be **stable**, when the beam after disturbance quickly resumes its original position of equilibrium. A balance would evidently be useless for weighing, if its equilibrium were *unstable* or even *neutral.*

For this, it is necessary that, when the scale-pans are equally loaded, the beam after displacement should come back rapidly to its position of equilibrium. Now in the inclined position of the beam

the only moment that tends to restore it to its original position is that of the weight of the beam.

For great stability, therefore, k must be large ; in other words, *the centre of gravity must be well below the fulcrum.*

It will be noticed that a balance is most sensitive when k is very small, and most stable when k is large ; thus the conditions of ***sensitiveness*** and ***quick weighing*** are to a certain extent antagonistic. In practice, this does not affect much, since the purpose, for which a balance is required, determines the relative importance between the two conditions. Thus for a balance used for ordinary commercial purposes such as in weighing coal, the main point is stability and rapid action ; for a balance used for research work in a laboratory quickness of weighing may be sacrificed for sensitiveness.

For an ordinary good balance in a laboratory, fair sensitiveness and reasonable rapid action can be secured by making k not very small and allowing the balance to have light and long arms.

82. Double Weighing.—In any case where the accuracy of a balance is doubted, either of the two following methods of ***double weighing*** is adopted :—

(*a*) **Borda's Method of Substitution.**—The body to be weighed is placed on the right-hand pan and is counterpoised exactly with fine sand or small shots placed in the opposite pan. Then the body is removed and replaced by weights from a weight-box until an exact balance is obtained. These weights are evidently equal to the required weight of the body, whether the balance is false or true, for they are placed in the same pan as the body and produce the same effect under exactly the same circumstances.

* See Intermediate Prac. Physics by the author.

(*b*) **Gauss' Method.**—The body is weighed in the ordinary way first in one pan and then in the other. If the two observed weights are equal, each is equal to the weight of the body, and the balance is true ; if not, the balance is false.

Let us assume that the balance is in exact equilibrium when there is no load on the pans and that it is false due to unequal lengths of the arms. If the pointer does not read zero, one of the scale pans is to be loaded with shots until it does.

Let a and b be the lengths of the arms and let a body whose true weight is W, appear to weigh W_1 and W_2 successively.

We have

$$W . a = W_1 . b \ldots (1)$$

and $$W_2 . a = W . b \ldots (2)$$

By multiplication,

$$W^2 . ab = W_1 . W_2 . ab$$

or $$W = \sqrt{W_1 . W_2}$$

i.e. the true weight W is the Geometric Mean of the observed weights W_1 and W_2.

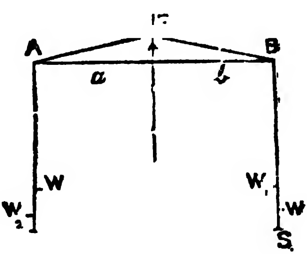


FIG. 98.
Double weighing.

Gauss' method is superior to Borda's for it is quicker and the two readings in it can act as a check to each other.

When, however, W_1 and W_2 are very nearly equal as they generally are, so that

$$W_1 = W_1 + \delta$$

where δ is small, sufficiently accurate result is obatined by taking the Amrithmetic Mean instead of the Geometric Mean of W_1 and W_2.

For $$\begin{aligned} W &= \sqrt{W_1 \times (W_1 + \delta)} \\ &= W_1 \sqrt{1 + \frac{\delta}{W_1}} \\ &= W_1 (1 + \tfrac{1}{2}\delta / W_1 + etc..) \\ &= W_1 + \delta / 2 \end{aligned}$$

The ratio of the lengths of the arms may be obtained from equations (1) and (2) above,

For

$$\frac{a}{b} = \frac{W_1}{W} \text{ and } \frac{a}{b} = \frac{W}{W_2}$$

$$\therefore \quad \frac{a}{b} = \sqrt{\frac{W_1 \times W}{W \times W_2}} = \sqrt{\frac{W_1}{W_2}}.$$

Again, if a tradesman uses a false balance having unequal arms of lengths a and b, and weighs out twice to a customer articles of apparent weight W by his balance by putting the weights first on one pan and then on the other, he really gives his customer $W_1 + W_2$.

$$\text{Now } W_1 + W_2 - 2W = W\frac{a}{b} + W\frac{b}{a} - 2W$$

$$= W\frac{a^2 + b^2 - 2ab}{ab}$$

$$= W.\frac{(a-b)^2}{ab} \text{ which is +ve always}$$

so long as a and b are unequal. It follows that

$$W_1 + W_2 > 2W$$

i e, the tradesman is a loser and loses by

$$W.\frac{(a-b)^2}{ab}.$$

Excercise—VIII.

1. A wheel and axle is used to raise a bucket from a well. The radius of the wheel is 15 ins., and while it makes seven revolutions, the bucket which weighs 30 lbs., rises $5\frac{1}{2}$ ft. Show what is the smallest force that can be employed to turn the wheel.—*Lond Matric.*

2. Find the inclination of a plane if the horizontal force of 5 kilograms' weight can just move a mass of 12 kilogrammes

3. A lever is 18 inches long. Where must the fulcrum be placed in order that a weight of 10 lbs., at one end may balance double its weight at the other end ?

4. A man whose weight is 200 lbs., is seated in a loop at one end of a rope passing over a smooth fixed pulley, and he holds the other end of the rope with both hands. Find the weight supported by each of his hands supposing that they support equal weights and that the two portions of the rope are parallel.

5. If there are six parts of the string at the lower block of a block and tackle, find the greatest weight which a man weighing 10 stones can possibly support.

6. A man raises a 4 ft cube of stone, weighing 2 tons, by means of a crow-bar, 3 ft. long, after having thurst one end of the bar under the stone to a distance of 6 inches : what force must be applied at the other end of the bar to raise the stone ?

7. In a pair of nut-crackers, 6 inches long, if the nut be placed at a distance of 1 inch from the hinge, a force equal to $3\frac{1}{2}$ lbs. wt. applied to the ends of the arms will crack the nut. What weight placed on the top of the nut will crack it ?

8. Describe with a sketch the balance you have used in your Laboratory.

What are the requisites of a good balance ? [*C. U.*—1922.

CHAPTER IX

PENDULUM

83. Vibration.—A motion is said to be **Periodic** when a particle or a body in motion continually returns to the same condition at regularly recurring intervals. The motion of a particle moving in a circle with uniform speed, the motion of a point in a vibrating string, as in a violin, are examples of periodic motion The motion of a planet round the sun is also periodic, for though the speed is not uniform, yet the velocity regains the same value at the end of a regularly recurring interval, for example a year in the case of the earth.

If, in addition to being periodic, the motion of a body is continually being reversed in direction, it is said to be **vibratory** or **oscillatory**. The motion of a pendulum in a clock, the motion of a tuning-fork, the up-and-down motion of a mass attached to a spring are instances of oscillatory periodic motion.

The vibrations or oscillations of a particle are mostly of the type of motion known as the *Simple Harmonic Motion*, often abbreviated into S. H. M.

The condition that is necessary for the Simple Harmonic Motion of a particle is that it must move in a staight line, so that its acceleration is always directed towards, and varies as its distance from a fixed point in the straight line, which indicates its final undisturbed position.

Def. of a S. H. M.

Also the force which tends to restore the displaced body to its original position varies as the acceleration (for $P = mf$), and is directly proportional to the displacement.*

84. Pendulum.—A *simple* pendulum consists of a heavy *particle* suspended by an inextensible string *without weight*, and oscillating *without friction* about a point to which the upper end of the string is attached. Such a pendulum is evidently not realisable in practice and is only necessary for a mathematical demonstration. A small sphere of lead or some other metal suspended by a thread so fine that its mass and weight may be negligible, forms a close approximation to a simple pendulum.

Any body which is supported in such a way that it can turn about a point or an axis is called a *Compound* or a *Physical* Pendulum One of the common forms consists of a metallic rod which turns about an axis at the upper end and carries at its lower end a heavy, lens-shaped mass of metal, called the *bob* which can be raised or lowered by means of a screw. The lens shape is preferred to a spherical form since, mass for mass, the resistance to its motion due to air is less in this shape so long as the thread is vertical. Now the weight of the particle is vertically below O, the point of suspension; so the pendulum, when

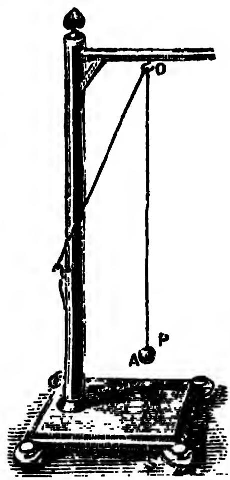


FIG. 99.
Pendulum.

* The Simple Harmonic Motion has been treated in a separate chapter in 'An Introduction to the study of *Sound*' by the author.

undisturbed, is in equlibrium. Suppose it is drawn aside to the position OB, making any angle θ with OA, the vertical direction. At B the weight mg of the mass m can be resolved in two directions. The component $mg \cos \theta$ acting along the string and in direction BD is, by the third law of motion, balanced by an equal and opposite force forming the tension T along the string; the other component $mg \sin \theta$ acting along the tangent BE to the arc at B is the effective force F to produce a downward motion of the particle. We have

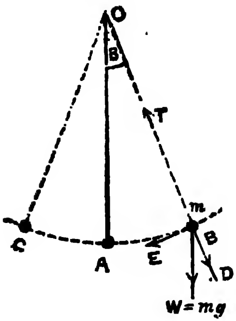


Fig. 100.
Pendulum.

$$F = mg \sin \theta$$

The bob, being urged by this force, will come down along an arc BA in a circle whose centre is at the point of suspension; the arc, however, continually diminishes with the angle of displacement and becomes zero when the bob arrives at the lowest point A. Now the bob cannot stop at A, but in virtue of its inertia and the acquired velocity it continues to move in the same directon on the opposite side of B. As the bob rises, however, the force due to gravity acting downwards now opposes the motion which accordingly becomes slower. Had there been no resistance of air and friction at the point of support, the bob would stop after it rises to a point C at the same height as B. It then descends again, passes through its mean position A, and returns to the point B. It will thus continue to oscillate between the two points B and C for an indefinite number of times, all the vibrations being of equal extent and performed in equal intervals of time.

In practice, however, the energy of the pendulum slowly diminishes, a part of it being continually spent in overcoming the resistances to motion. The effect is that the amplitude of its oscillation gradually diminishes until the whole stock of its energy is exhausted, when it comes to stop at its initial position of rest at *A*.

The maximum distance AB or AC through which the bob A of a pendulum is displaced on either side of its equilibrium position O, is called its **Amplitude.**

85. Period.—The period of a pendulum is the time which it takes in moving from B to A and back to B or more generally is the time from its passing through a given position to its next passing through the same position in *the same direction.*

*In art. 84 it has been seen that

$$F = mg \sin \theta$$

where F is the effective component of the weight mg of a bob of mass m, when the bob has been displaced through an angle θ on one side. If θ is very small, sin θ may be put equal to θ.

We have then

$$\text{Effective Force} = mg.\ \theta$$
$$= mg.\ \frac{\text{arc AB}}{l}$$

$\therefore$ acceleration of the bob at the instant under consideration

$$= \frac{g}{l} \times \text{displacement AB.} \qquad \ldots (1)$$

In other words, the acceleration along the tangent to the path varies as the displacement. It follows that the motion is harmonic for small ranges of vibration. It may be shown that t the time of a complete oscillation is given by

$$t = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{l/g} \qquad \textit{from } (1)$$

and that it is independent of the extent of the oscillation when θ is small and is calculated to be less than about 4°. This formula sums up all the laws of the simple pendulum for oscillations through a small amplitude.

86. Laws of Pendulum.—The oscillations of a pendulum are expressed by the following laws :—

Law 1, Law of Isochronism—The oscillations of a pendulum are isochronous, *i.e.*, effected in equal times provided the amplitudes are small. This law is perfectly exact when the angle of displacement is not more than 4°; for greater amplitudes, the oscillation is longer. The uniform rate of motion of a clock depends on this property of a pendulum.

Expt. 35. To verify this, note by means of a stop-watch the total time taken by a pendulum to perform some 20 complete oscillations with different amplitudes. It will be found that the time taken in each case remains the same.

The law was first discovered by the celebrated physicist and astronomer GALILEO (1564-1642). This was the very first discovery of his and this he made before he was twenty years of age, and while he was still a student of medicine at the University of Pisa. He happened one day to observe in the cathedral at Pisa the swinging of a bronze lamp hanging from the lofty roof, and was struck by the fact that the oscillations of the lamp, whether of smaller or greater extent, appeared to occupy equal intervals of time. To make quite sure of this, he put his fingers on his own pulse and comparing its throbs with each swing of the lamp, found that there was always the same number of beats to every swing. Following up this simple observation he discovered that a weight at the end of a cord will always take the same time to swing backwards and forwards, so long as the cord is of the same length and the arc through which the weight moves is small. This was the *beginning* of pendulums, though at first they were only used by physicians to count the rate at which a patient's pulse beats. The merit of having first made the application of a pendulum to clocks is generally attributed to Huyghens (1629-1695).

Galileo's discovery of the Principle of Pendulum. (1581)

Galileo also discovered the law of length which is given below.

Law II. The Law of Length.—The period of oscillation of a simple pendulum varies as the square root of the length, *i.e*, $t \propto \sqrt{l}$ Thus if the length of a pendulum is increased 4, 9 or 16 times, the period will be 2, 3 or 4 times respectively,

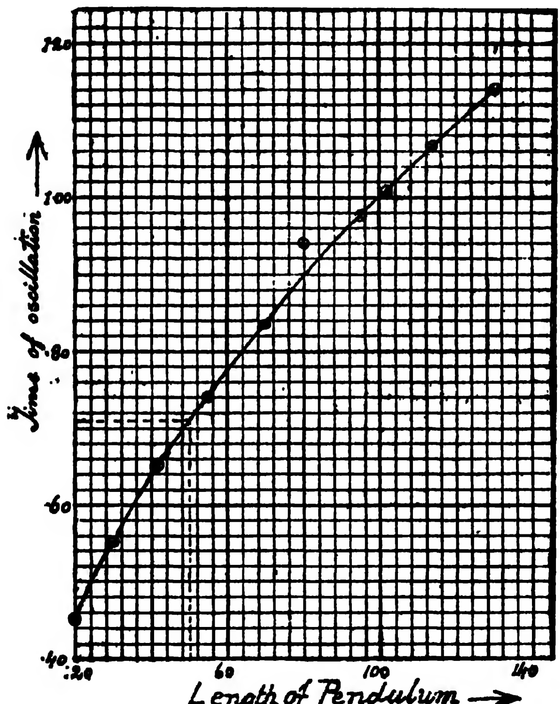


FIG. 101.

L—T Graph for a pendulum.

The length of a simple pendulum is the length of the supension thread from O, the point of suspension, to A, the centre of oscillation. In a physical pendulum

this is most approximately given by the distance measured from the point of suspension to the centre of gravity of the bob, for example, to the centre of the sphere, when the bob is a spherical body.

Expt. 36. Measure with a slide-calipers the diameter of the spherical bob of a pendulum and hence find its radius. Use a metre scale to measure the length of the pendulum. Observe by means of a stop-watch the time taken by the pendulum to perform 20 complete oscillations. Hence find t, the period

Make similar observations when the length of the pendulum is changed to different values. Tabulate your observations so that one column contains the observed periods, while a second column contains the square roots of the corresponding lengths. It will be found that $t/\sqrt{l}$ is constant.

L—T Graph

If a graph be drawn to represent the relation between the length and the time from a set of observations on the oscillations of a pendulum †, it will be of the nature of a parabola (fig 101).

When a clock is going too slow or too fast, the length of its pendulum must be altered to regulate it. If the clock goes too fast, it means that its pendulum oscillates too quickly and the latter must therefore be lengthened. This is secured by lowering the lens-shaped bob a little along the pendulum rod.

Law III.—The period of oscillation of a simple pendulum is independent of the mass and the material of the bob. In other words, if the length of the pendulum remains the same, it does not matter whether the bob is of wood, or brass, or ivory or some other material; the time of swing will remain the same.

Expt. 37. Find the periods for bobs of different materials. Adjust the length of the string so that the length of the pendulum *i.e.*, the distance between the point of suspension and the centre of the ball, remains the same. Time will be found to be unaltered.

† See Intermediate Practical Physics by the author. For the data see *Q. 5. Exercise IX.*

If in this experiment, the time for a large number of oscillations be observed, the period of oscillation for spheres of lighter materials will be found to increase a little This is due to the resistance of the air which has more effect on the lighter than on the heavier bodies.

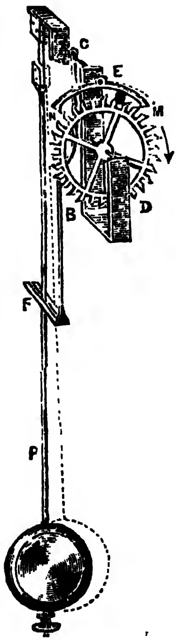


FIG. 102.

The escapement of a clock pendulum.

Law IV.—The period of oscillation varies inversely as the square root of the acceleration due to gravity at the place where it oscillates *i.e.*, $t \propto 1/\sqrt{g}$

It follows that if the value of g at a place is greater than that at another place, the period of oscillation of a pendulum will be smaller, *i.e.*, the vibrations will be quicker.

87. Application of the Pendulum to Clocks—Huyghens in 1658 first applied the pendulum to regulate the motion of clocks and in the same year Hooke applied a spiral spring to the balance-wheel of a watch. In fig. 102. the pendulum rod P, passing between the prongs of a fork F, communicates its motion to a rod B, fixed above to a horizontal axis at To this axis is fixed the *escapement* E, having two projections or *pallets*, working alternately with the teeth of the wheel D which is urged in the direction of the arrow either by a clock-weight or by a wound spring. As the pendulum oscillates, and takes the position shown by the dotted line, the pallet *M* of the escapement is raised and the wheel D escapes As it turns through, its motion is arrested by the other pallet *n*, coming into contact with another tooth of the wheel due to the motion of the pendulum Thus the motion of the wheel

alternately permitted and arrested *i.e.*, *regulated* by the pendulum. Then by means of a suitable train of wheelwork, the motion of the wheel is communicated to the hands of a clock. It is to be noticed also that the slight pushes communicated by the teeth of the wheel to keep the pendulum from dying down.

88. The Value of 'g' by Pendulum.—The formula for the period of an oscillation of a simple pendulum provides an excellent and the best means of determining accurately the value of 'g', the acceleration due to gravity at a place. If we observe t and l, we have

$$t = 2\pi\sqrt{\frac{l}{g}}$$

whence $$g = \frac{4\pi^2 l}{t^2}$$

and from this we can calculate 'g'.

The pendulum experiments have established that while g is a constant for all bodies at a given place on the earth's surface, it varies from place to place; in other words, the value of 'g' varies with the latitude. It increases as one proceeds from the equator to either of the poles. It is about 978 cm. per sec. per sec. at the equator and about 983 cm. per sec. per sec at the pole. This diminution of g from the pole to the equator has already been considered (art 54).

The acceleration of gravity is found from pendulum experiments also to diminish as we ascend higher above the sea-level, or descend into the earth as in a mine. It may be theoretically shown that for any particle *outside* a spherical *shell*, the attraction it exerts is the same as if its whole mass is concentrated at its centre; while for a point inside it, the force is nil. Hence for a place at a height h above the sea-level the gravitative force F exerted by the earth on a [illegible] cle mass m is expressed by

$$F \propto \frac{Mm}{(R+h)^2}$$

where M is the mass of the earth and R, its radius. The acceleration g_1, therefore, of particle m is given by

$$g_1 \propto \frac{1}{(R+h)^2} \quad \text{or} \quad g_1 = K \frac{1}{(R+h)^2}$$

where K is a constant ; while at the surface

$$g \propto \frac{1}{R^2} \quad \text{or} \quad g = K \frac{1}{R^2} \quad ;$$

so that $$g_1 = g \left(\frac{R}{R+h}\right)^2$$

i e., g decreases as we ascend higher. It follows that the period of oscillation of a pendulum will increase.

At any place *inside* the earth, the acceleration due to gravity at a depth d must be less than what it is at the surface, for obviously there will now be some portion of the earth which will exert an attraction on any body at the place in a direction opposite to that in which the rest of the earth pulls the body ; so the acceleration will be zero at the centre of the earth. Theoretically it may be explained thus : let a spherical surface be drawn through a point P at the depth d, round O, the centre of the earth. Then the point P is an *inside* point with respect to all the shells of the earth that are lying between this surface drawn through P and the surface of the earth ; hence their attraction on a mass at P is nil. The rest of the earth, enclosed between its centre and the surface through P, for which P may be regarded as an outside point, exerts an attractive force on the mass at P ; this may be shown to vary directly as the distance OP. Hence the acceleration inside is given by

$$g_2 \propto R-d \qquad \text{or } g_2 = k\,(R-d)$$

where R is the radius of the earth and k is a constant ; while the acceleration on the surface may be written as

$$g = k.R$$

so that $$g_2 = g \times \frac{R-d}{R}$$

89. Seconds Pendulum.—A simple pendulum which vibrates from rest to rest *i. e*, makes half a complete oscillation in one second, is called a **Seconds Pendulum.** Hence from the formula for the period of oscillation we have

$$1=\pi\sqrt{\frac{l}{g}}$$

$$\therefore \qquad l=\frac{g}{\pi^2}$$

Since g varies at different places on the earth's surface, the length of the seconds pendulum is not also the same for all places. For an approximate value, putting $g=32\cdot2$ ft. per sec per sec. and $\pi^2=9\cdot87$, we have

$$l=3\cdot26 \text{ ft.}=39\cdot12 \text{ inches.}$$

and putting $g=981$ cm. per sec. per sec. we get

$$l=99\cdot39 \text{ cm.}$$

EXAMPLES :—

1. A faulty seconds pendulum loses 9 seconds per day; find the required alteration in its length, so that it may keep correct time?

1 day = 86400 seconds.

Since the pendulum loses 9 seconds per day, it beats (86400 − 9) or 86391 times per day *i.e.*, in 86400 seconds; so that its time of oscillation is 86400/86391 second (and not 1 second as it ought to be). Let l be its length

$$\sqrt{\frac{l}{g}}=\frac{86400}{86391} \qquad \ldots \quad (1)$$

Let its length be changed from l to $l+x$ to make it keep correct time; since, in that case, it becomes a true seconds pendulum, its time of oscillation becomes 1 second. Hence

$$\pi\sqrt{\frac{l+x}{g}}=1. \qquad \ldots \quad (2)$$

From (1) and (2)

$$\pi^2\frac{l}{g}=\left(\frac{86400}{86391}\right)^2 \text{ and } \pi^2\frac{l+x}{g}$$

Subtracting

$$\pi^2 \frac{x}{g} = 1 - \left(\frac{86400}{86391}\right)^2$$

$$= 1 - \left\{ 1 + \frac{9}{86391} \right\}^2$$

$$= 1 - \left\{ 1 + \frac{18}{86391} + \text{etc.} \right\}$$

$$\therefore -\frac{18}{86391}$$

$$\therefore x = \frac{g}{\pi^2} \times \left(- \frac{18}{86391} \right) = - \frac{32 \times 7^2}{22^2} \times \frac{18}{86391}$$

$$= - \cdot 008 \text{ m.}$$

Hence the length of the pendulum must be *shortened* by ·008 in.

Exercise.—IX.

1. Two simple pendulums of length, 1 metre and 1·1 metre respectively start swinging together with the same amplitude. Find the number of swings that will be executed by the longer pendulum before they are again swinging together ($g = 978$). [*C. U.—1909.*

2. Describe in detail how you would test by means of pendulum experiments whether the acceleration due to gravity is the same for all substances. [*C. U.—1910.*

3. State the laws of oscillation of a simple seconds pendulum Find the length of a simple seconds pendulum at a place where g is 981.

When a ball suspended by a string is made into a 'seconds pendulum' does the actual length of its string equal the length of the equivalent simple pendulum? If not, why? [*C. U.—1912.*

4. State the laws of oscillation of a simple pendulum. Describe the effect of temperature on the period of oscillation of a compound pendulum. [*C. U.—1913.*

5. State the laws of the pendulum.

The following readings were obtained with a si pendulum ;—

Length	Time of oscillation	Length	Time of oscillation
20 cm.	·45 sec.	80 cms.	·94 sec.
30 ,,	·55 ,,	95 ,,	·98 ,,
42 ,,	·65 ,,	102 ,,	1·01 ,,
55 ,,	·74 ,,	115 ,,	1·07 ,,
70 ,,	·835 ,,	130 ,,	1·14 ,,

Represent by a graph the relation between length and time, and find from your graph the time of oscillation of a simple pendulum of length 50 cm. [*C. U.* –*1915*.

6. The laws of the simple pendulum are summarized in the formula $t = 2\pi\sqrt{l/g}$. Explain clearly the meaning of each symbol in the formula.

If the frequency of oscillation of a pendulum in 98 per minute at a place where g=980 cm. per sec. per sec, find the length of the pendulum. [*C. U.*–*1916*.

7. What is a Simple Pendulum ? Find the length of the Seconds Pendulum at a place at which g=981.

What is the exact meaning of the statement g=981 ?

Will a pendulum clock gain or lose when taken to the top of a mountain from the bottom ? [*C. U.*—*1917* ; '*19*.

8. State the laws of simple pendulum How will you proceed to determine the 'g' of a place with a pendulum ? Give the practical directions necessary and state reasons.

What is the effect of the height above, or the depth below the surface of the earth, on the periodic time of a pendulum ? Explain. [*C. U.*—*1921*.

9. State the laws of the pendulum. Will the period of vibration of a pendulum be affected if it be taken to the top of a hill ? Give reasons for your answer. [*C. U.* - *1924*.

10. A faulty seconds pendulum loses 20 secs. per day. Find the alteration in length so that it may keep correct time.

CHAPTER X.

WORK AND ENERGY

90 **Work.**—When the point of a body at which a force acts, is displaced in the direction in which the force acts, **work** is said to be done *by* the force.

A horse drawing a cart along a rough level road does work; an engine drawing a train does work. In each case, the body pulled on moves in the direction in which the force, as a pull, is exerted. Here work is done *by* a force.

Work by a force.

But at the same time work is done also *against* some other force. Thus when a horse draws a cart, work is done against the frictional resistances of the ground; in fact, had the ground been perfectly smooth, no work would have been done in drawing a body along its surface; similarly, when a heavy mass is lifted up from the ground, work must be done against its weight.

Work against a force.

The work done *by* a force is measured by the product of the magnitude of the force and the displacement measured along the line of the force.

Measurement of work.

Let s be the displacement of the point of application of a force F in the direction of action of the force, then W, the work done *by* or *against* the force is given by

$$W = F.s$$

When the displacement s takes place in a direction making an angle θ with the direction of the force, the component of the displacement in the direction of the force is $s \cos \theta$. The work done by the force is given by

$$W = F s \cos \theta$$

No displacement, no work.

It should be noticed that no work is done by a force in a direction at right angles to that of the force, for a force has no component in a direction perpendicular to its own line of action. Also no work is done, when there is no displacement of the point at which the force acts Thus when a man is unsuccessful in lifting a heavy weight, however hard he may try to do it, he does *no* work against the force of gravity.

Work and Time.

Moreover, in the expression for work, the time in which the displacement takes place, does not occur. Hence work done is the same whether a given displacement is suffered with greater or smaller velocity.

Power.

From a practical point of view, however, it is important to consider not only the amount of work done by a machine but the time taken by it to do the work; in other words, the rate at which it is done, which is called the **Power** or activity.

91. Units of Work.—The unit of work is the work done by a *unit* force acting through a *unit* distance, whatever system of units be chosen.

In the C. G. S. system the unit of work is the work done when a force of *one dyne* acts through a distance of a centimetre. This is called an **Erg.**

In the F. P. S. system the unit of work is the work done when a force of *one poundal* acts through a distance of one *foot.* This is called a **Foot-poundal.**

Work is very commonly expressed in gravitational units. The *gravitational unit of work* is the work done in raising a unit mass *against gravity* through a unit distance. In the English system the unit which is called a **foot-pound** is the work done in lifting a mass of 1 lb. vertically through 1 ft.; since the weight of 1 lb. is g poundals (art. 55), g being the acceleration due to gravity at a place

$$1 \text{ ft. pound} = g \text{ foot-poundals}$$
$$= 32\cdot2 \text{ (average value) ft-poundals.}$$

In the metric system the corresponding unit is the **gramme-centimetre** or the work done in raising 1 gram through 1 cm. Since 1 gram equals g dynes.

$$1 \text{ gram-cantimetre} = `g' \text{ ergs}$$
$$= 981 \text{ (average value) ergs.}$$

Thus it follows that the work done in lifting a mass M gms. through h centimetres is

$$= Mh \text{ gramme-centimetres}$$
$$= Mhg \text{ ergs.}$$

The erg is a very small quantity; for example, the work done in lifting a pound through 1 ft.,—in other words,—the foot-pound

$$= 32\cdot2 \text{ foot-poundals}$$
$$= 32\cdot2 \times 30\cdot48 \times 13780 \text{ ergs}$$
$$\text{since } 1 \text{ ft.} = 30\cdot48 \text{ cm}$$
$$\text{and } 1 \text{ poundal} = 13780 \text{ dynes}$$
$$= \text{about } 13\cdot6 \text{ million ergs.}$$

A larger unit, called a **Joule** is employed for industrial purposes and is equal to $4\cdot2 \times 10$ million $(10)^7$ ergs.

Hence

$$\text{a foot-pound} = 1\cdot36 \text{ joules.}$$

92 Energy.—We find by experience that in certain circumstances bodies are capable of doing work. The **energy** of a body is its capacity for doing work.

A body may possess energy in virtue of its being in *motion.* Thus every moving body is capable of working against resistances until it comes to rest. For example, the flying bullet, when it strikes a wooden target, penetrates a considerable distance into the wood working against the cohesive forces between the wood particles. A running stream is able to turn the wheel of a water-mill, the energy of which may be utilized in working grinding-machines, electrical generators (dynamos) and various other machines. The energy of the wind in motion pressing against the sails of a boat may drive her in motion which would thus overcome the resistance through water offered to its passage

Energy of motion or Kinetic Energy.

This form of mechanical energy is called the **Kinetic Energy.** Wherever we find matter in motion, be it solid, liquid or gaseous, it possesses kinetic energy. Further, the motion of a body may be a *translatory* motion (as in the case of a falling body, a shot fired from a gun etc.), or a *rotatory* motion (as in the case of a spinning top) or a *vibratory* motion (as in the case of a vibrating pendulum, the vibrating particles of a sounding body etc.), or a motion of any other kind.

A body may also possess energy in virtue of its *position* relative to a body which attracts it. Thus when a weight has been raised up above the ground, it possesses energy due to its elevated position ; all the work spent on it in lifting it up is stored up in it, which is ready to be freed whenever the body shall be permitted to fall. For example, while falling, it can pull up a lighter body attached to it by means of a string passing over a pulley. In winding a clock, driven by a falling weight, work is spent on the weight to lift it up. As long as the weight remains at the elevated position, its energy is stored up, which will be expended during its fall in driving the clock and in overcoming the friction

Energy of position or Potential Energy.

of the machinery. This form of mechanical energy is called **Potential Energy.**

Measurement of energy.

Energy is measured as work; for when a body *does work against a force*, it loses energy, and if work *is done on it* by an external force, it gains energy. In either case, the loss or gain of energy is measured by the *work done by or against a force*. Hence the units employed in the measurement of energy are the same as those of work. It will be noticed that in the measurement of energy we are concerned with the measurement of that which a body or a system of bodies *gains or loses*, and not the whole energy which such boby or bodies, may possess.

93. Kinetic Energy.—As has been stated in the last article the kinetic energy of a body is the energy which a body possesses in virtue of its motion. Instances of bodies possessing kinetic energy have already been given in art. 92.

Let us find an expression for the kinetic energy of a moving body :—

Suppose a body of mass m to be moving with velocity u. We have to find the work it is capable of doing before it comes to rest.

Let the velocity change from u to v under the action of a constant resisting force P which produces in it an acceleration of f, we have $P=mf$.

Let s be the space passed over before the body comes to rest, so that from the equation,

$$v^2=u^2+2fs$$

We have $\quad 0=u^2-2fs$

Or $\quad u^2=2fs$

Hence the kinetic energy of the mass

= work done *by* the body *against* the force

= force × distance

$= p.s = mfs = \frac{1}{2}mu^2$

Thus the kinetic energy of a body is half the product of its mass into the square of its velocity.

If the force P change the velocity of the mass m moving without rotation from u to v in passing over a distance s, we have

$$v^2 = u^2 + 2fs$$

$$\text{Or} \quad v^2 - u^2 = 2fs$$

The work done by the force

$$\begin{aligned} &= P.s. \\ &= mf\frac{v^2 - u^2}{2f} \\ &= \tfrac{1}{2} m (v^2 - u^2) \\ &= \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2 \\ &= \text{final } K.E. - \text{initial } K.E. \\ &= \text{increase of } K.E. \text{ of the body.} \end{aligned}$$

In particular, if the body starts from rest, $u=0$; the final gain of $K.E.$ of the body is equal to the work done by the force

$$= \tfrac{1}{2} mv^2$$

Similarly, when the velocity of the mass decreases from u to v, the *loss* in $K.E.$ is given by

$$\tfrac{1}{2} mu^2 - \tfrac{1}{2} mv^2$$

When the force is variable, we may divide the time for which the force acts into very small parts, during which the force may be supposed not to alter. We may then apply the last formula to each part, and find, by addition, the total work done in a given time.

If m is expressed in grams and v in centimeters per second, the $K.E.$ is expressed in ergs. Similarly, when m is expressed in pounds and v in feet per sec., the $K.E.$ is expressed in foot-poundals.

As $K.E.$ is proportional to the square of v, it follows that the effect of collision or derailing in the case of an express or a mail train is much more destructive than that in the case of a goods train though the mass of the latter is much greater.

94. Potential Energy—Some instances of bodies possessing this kind of energy have already been given in art. 92. The Potential Energy may be defined to be *the energy possessed by a body (or a*

ystem of bodies) in virtue of its position or configuration (i. e. the position of its parts). It is measured by the work which a body is capable of doing, in passing from its present position to some standard position; or what is the same thing to say, by the amount of work done upon the body by a force in order to bring it from a standard position to its present position.

Measurement of pot. E.

Thus when a spiral spring is compressed or pulled out, work has to be done against the elastic resistance offered by the spring to its disfiguement: this work is stored up as potential energy in the spring. If the spring be allowed to regain its previous form, it is able to do work against an opposing external force and so loses energy. The spring gains and loses energy by the change of configuration.

Pot. E. due to ch. in configuration or *strain.*

In the same way a bent elastic strip of metal, a coiled watch-spring, an extended india-rubber cord, an excited violin string, an extended bow string, a quantity of compressed air, all possess potential energy due to *strain* or deformation in their materials, and all can do work when allowed to pass from their present configuration to the initial one. Strain energy is thus a form of potential energy.

The most important case of potential energy is, however, what a body possesses when it is lifted up from the surface of the earth. As a mass is raised upwards, work is done against its weight and the body gains potential energy in virtue of its change of position relative to the earth It is usually termed the *Gravitational Potential Energy* of the body. Water stored up in an elevated tank, a raised clock-weight, and an uplifted hammer, water-vapour floating in the air all possess gravitational potential energy.

Pot E due to change in position.

It would have been proper and accurate, however, to regard the earth and the body on or near its surface as forming *one system*, and that the system, made up of the earth and the body gains a quantity of energy equal to the work done in raising the body against the mutual attractions of the two, for the mass itself in the absence of the earth can have no potential energy.

Gravitational Pot. E. = mgh.

It has been shown in art. 91 that if a body of mass m is raised through a *vertical* distance h at any place where g is the acceleration due to gravity, the work done against its weight is mgh. Therefore the gravitational potential energy gained by the mass in this operation is mgh. Conversely, when a body of mass m falls through a vertical distance h at any place, work done *by* its weight in falling is mgh which measures the *loss* of potential energy of the body.

It should be noticed that h in the expression mgh is the *vertical displacement* of a body and may not be its actual path of displacement.

Magnetic Pot. Energy.

Potential energy may also be stored by doing work against magnetic and electric forces. A magnet attracts a piece of iron which tends to stick to the former. Hence when they are separated work is done and the system, consisting of the magnet and iron, acquires potential energy.

When a body is in stable equilibrium, its centre of gravity occupies the lowest position possible to the body (art 64). Any displacement tends to raise the centre of gravity of the body. Hence every such displacement tends to increase the potential energy of the body. Therefore every state of stable equilibrium of a system (free from frictional resistances) corresponds to a *minimum* of *potential energy*.

95. Forms of Energy.—The energy which a body or a system of bodies possesses may exist in

many forms but can always be put into either of the two classes, kinetic or potential or a combination of these two. The different forms in which energy may appear are :—

1. MECHANICAL ENERGY : *e.g.*, the energy of a body falling from a height, of a body in vibratory motion etc.
2. HEAT : the molecular kinetic energy in a body.
3. SOUND : the energy of longitudinal wave-motion in a material medium.
4. LIGHT : the energy of transverse wave-motion in ether.
5. ELECTRICAL and MAGNETIC Energy.
6. CHEMICAL Energy.

Frequently however, one form of energy is changing into another form. As a matter of fact all the phenomena in the universe are but cases of transformation of energy, which are almost endless in their variety.

96. Transformation of Energy.—In this article we shall consider some of the cases of transformation of energy.

When a body is at a height above the ground, it possesses gravitational potential energy. When it is allowed to fall, it gradually loses potential energy. The energy thus lost to the body is partly expended in overcoming the frictional forces of the air and partly in imparting kinetic energy to the body due to which the velocity is gradually increased. Just before striking the ground all the energy that the body possesses is kinetic. As the ball strikes the ground, its energy is transformed into heat (within the ball

Falling body.

and the ground), sound and the mechanical energy of rebound, the latter two forms finally dissipating into heat in the surrounding space.

The energy of a body in *vibratory motion e. g.*, an oscillating pendulum bob, a vibrating violin string etc., is a combination of kinetic energy and potential energy. The energy of vibration of a pendulum bob at any instant is what the bob possesses at that instant during its vibration, *in excess of* the energy it possesses when at rest. When the bob is at either of the extreme points, B or C in its path of vibration (fig. 100), the energy of vibration is wholly gravitational potential energy; and when the bob is at its initial position A, the lowest point in its path, the energy is wholly kinetic; at any other point in its path, the energy is partly kinetic and partly potential. As the bob proceeds towards the lowest point, it falls in height and therefore loses potential energy but gains kinetic energy; conversely as it passes from the lowest point towards either of the highest points, it loses kinetic energy and gains potential energy. The total energy of the bob may be proved to remain constant excepting what is expended in overcoming air resistance and also to a certain extent the friction in the suspension thread at the point where bending takes place. Due to this reason the amplitude of vibration decreases gradually until the pendulum comes to rest. In order that the pendulum of a clock may continue in motion, energy is to be regularly supplied by the strain energy of a clock-spring of the potential energy of a raised clock weight.

Vibratory Motion,— A pendulum.

In the *molecular theory* of matter a piece of matter may be considered as an aggregate of molecules held together by the action of the intermolecular forces (art. 102). A body may, therefore, possess **molecular potential energy** in virtue of its molecular

Molecular Energy.

configuration, *i.e.*, the relative positions of the molecules to each other. The ***strain*** energy or the energy due to the deformation in a body is thus the gain in its molecular potential energy. It is also supposed that the molecules in a body are not at rest but in extremely minute and rapid vibratory motion, so that the body possesses **molecular kinetic energy** also.

Heat of a body.

The molecular kinetic energy of a body is associated with the **heat** of a body, which raises its temperature. When a button is rubbed on a table-cloth and then applied to the skin, it is found to be appreciably hot. The kinetic energy of the button, lost as such, reappears in the new form of energy as heat. Conversely, appearance of kinetic energy is frequently accompanied by loss of heat. For example, the steam cools in moving the piston in a steam-engine, a gas cools when allowed to expand suddenly.

Change of state.

Again when a body changes its state *i. e.*, passes from the solid to the liquid form or from the liquid to the vapour form, it always absorbs a quantity of heat energy which is called the **Latent Heat** as it is not indicated in the rise of temperature of the body. This heat-energy is all expended in increasing the molecular potential energy that is necessary to bring about a change of state.

Wave-motion, —Light.

Light is believed to be transverse wave-motion in the medium ether, through which it passes with a definite velocity, and on reaching our eyes affects our sense of sight. The particle of a medium through which a wave-motion is propagated are set into vibratory motion ; hence energy of a medium is a mixture of kinetic and potential energies. It has been calculated that to affect us as light the waves must be of lengths, within certain narrow limits, somewhere between 300 and 800 millionth of a millimetre.

But as we are familiar with the heating effect in the light from a glowing fire or from the sun, we are to infer that incandescent bodies send out from them other waves also, similar in kind to light waves, but being of length outside the above limits they are incapable of exciting our sense of sight. The whole series of waves coming out is given the general name of **Radiation** : the corresponding energy that is conveyed is called the *Radiant Energy*. Radiant Energy is thus the form into which the heat-energy in the radiating body changes when it is transferred into the surrounding medium as wave-motion.

Radiation.

The energy of a tuning-fork or of a stretched wire set into vibration is similar to that of a pendulum bob. As the body vibrates, there is a periodic transformation of *strain* and *kinetic* energy. The energy of the vibrating body is exhausted in giving rise to sound-energy in the surrounding medium and heat energy in the body to overcome internal friction.

Sound.

If a rod of ebonite is rubbed with flannel, it seems to acquire a new property of attracting bodies such as bits of paper towards it. The new form of energy is called **Electricity**. Similarly when the blade of a steel-knife is rubbed with one pole of magnet, it is found to acquire the new property of attracting small pieces of iron ; the form in which the energy reappears is called **Magnetic energy**. In both the cases some amount of kinetic energy has to be spent. These two forms of energy are closely associated with each other and are regarded to be manifestations of different forms of energy of *strain and motion in the ether*.

Electricity.

Magnetism.

A molecule of a body may be supposed to be an aggregate of atoms held together by *chemical affinity*. When water is decomposed by electric current into the constituent elements, hydrogen and oxygen, the

latter possess energy of chemical separation against atomic attraction, in other words, potential **chemical energy,** the case being analogous to the energy of separation of a mass from the earth against the attraction of gravity. A small flame or an electric spark will suffice to rapidly convert the chemical energy of a mixture of hydrogen and oxygen into the kinetic energy of explosion. A loaded cartridge has a store of potential chemical energy which can, at any moment, be converted into the kinetic energy of the bullet.

Chemical energy,

When a person lifts up a heavy weight by muscular exertion, the muscular tissue concerned in the action undergoes partial decomposition. The chemical energy thus lost by the tissue is transformed into other forms of energy Thus a part appears as heat-energy in the muscular portion and a part is expended in doing work on the body, which is stored up as gravitational potential energy in the body.

Muscular energy.

Solar energy enables plants and trees to decompose carbon dioxide present in the air and to grow by absorbing carbon portion. Coal which is our chief fuel consists of carbon of wood which grew thousands of years ago and was then subjected to great pressure under the earth. Its combustion means the re-union of carbon and oxygen atoms long separated. Thus coal and wood are stores of potential chemical energy produced by sun's light and heat. When they burn, their chemical energy is reconverted into heat and light.

Energy of Coal.

The heat-energy obtained by the combustion of coal may again be changed by a locomotive steam-engine into the kinetic energy of the piston, of the driving wheel and then of the moving train. As mentioned before, a part of the mechanical energy reappears as heat in the various parts wherever friction is overcome. In the case of a stationary-engine used in driving a dynamo, the heat expended by the engine may be transformed into electri-

al energy which again may be utilized to do mechanical vork as in driving a tram-car or a fan, or to produce ight as in electric lamps. or to produce heat as in lectric stoves and radiators.

To take another example of transformation of energy, ast quantities of sea-water are being daily evaporated under the action of the sun's rays, the vapours so formed, condensing into clouds in the higher regions of tmosphere. This water comes down to the earth in he form of rain which feeds the running streams and lows down to the sea, its place of birth. The energy f the running stream is thus derived from the potental nergy of the clouds, of which again the source is the olar energy.

Energy of running stream.

97. **Conservation of Enery.**—The examples onsidered in the last article point to the fact that the isappearance of one of the forms of energy is always ccompanied of the appearance of one or more of the ther forms. Indeed, the results of many experiments nade with the greatest care have led to the conclusions hat there can be no case in which one kind of energy a absolutely annihilated without the appearance of nother, nor any in which a form of energy appears *de novo* without the loss of another.

Thus, a body or a system of bodies, may lose nergy in one form, and gain an equal amount of energy in some other form. Or a body or a system of bodies A may do work on some other body or system of bodies B, and thereby lose energy in some form, and the body r the system of bodies B gains energy either in the same orm or in some other form or forms, and the energy ost by A is equal to that gained by B. If A and B re regarded together, it is evident that the total mount of energy in the complete system must emain constant and cannot be increased or diminished n any way.

Conservation f Energy.

This is known as the Principle of **Conservation of Energy**, and is a fundamental law in Physics. Maxwell states this principle thus :—

The total energy of any material system can neither be increased nor diminished by any action between the parts of the system though it may be transformed into any of the forms of which energy is susceptible.

It should be noticed that the statement "by any action between the parts of the system" implies that the system is to be regarded as isolated from the action of all bodies external to itself. It follows that the total quantity of energy present in the universe always remains the same.

It is also to be remembered that in any case where energy is transformed from one form to another it does so according to a definite rate of exchange. Thus it has been established that the quantity of heat necessary to raise the temperature of 1 lb. of water through 1° F possesses the same amount of energy as is required to lift 772 lbs. against gravity through one foot *i.e.*, the mechanical equivalent of the unit quantity of heat is 778 foot-pounds.

It follows from the principle of Conservation of Energy that the total quantity of enrgy in the universe is constant.

98. Dissipation of Energy.—Although the total quantity of energy in the universe remains constant, so that the disappearance of energy in one form is always accompanied by the appearance of energy in some other form or forms, energy available to man for the purpose of doing work is continually diminishing. To take an example, if a stone dropped from a great height, strikes the ground, its kinetic energy disappears ; and the stone and the earth become warmer as the result of impact. In a very short time the heat of the stone will be diffused among the surrounding objects. So far as we are concerned, the energy of the stone has been wasted.

In a heat-engine a large proportion of heat produced by the combustion of the fuel is lost in doing work against friction within the machinery and is thereby converted into heat which warms up these parts.

In fact, in every transformation of energy, a part of of the energy becomes converted into uniformly diffused heat Energy in this form is of no more use to us for doing work Heat can be expended in doing work only when it passes from a hotter booy to a colder one, for example, from the boiler to the condenser in a steam-engine. If the temperature of a system is uniform throughout, no portion of the energy of the system is available for use.

As the total quantity of energy in the universe is a constant quantity, it follows from the above that the stock of energy available for work is steadily decreasing and that there must arrive a time when all the energy will be unavailable, the whole universe having become uniformly hot and inert mass.

EXAMPLES :—

1. A piston is moved along a cylinder against a constant pressure p. Find an expression for the work done.

Let the sectional area of the piston $=a$

And the displacement of the piston $=l$

Then work done $=$ force $\times$ displacement

$$= pa \times l \text{ ergs}$$
$$= p \times al$$
$$= p \times v,$$

where v is the volume through which displacement has taken place.

$$\therefore \quad W = pv \text{ ergs}$$

2 Prove that when a particle of mass m falls from rest at a height h above the ground, the sum of its potential and kinetic energies is constant throughout the motion,—if the frictional resistance due to air be neglected.

Let v be the velocity of the particle when it has fallen through any distance x.

We have $v^2 = 2gx$.

Its kinetic energy at that instant is given by

$$\text{K. E} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m \times 2gx = mgx$$

Also its potential energy at this height.

$$= mg\,(h - x)$$

Hence the sum of its kinetic and potential energies

$$= mgx + mg\,(h - x) = mgh.$$

which is the potential energy of the particle at the height h forming its total energy.

Exercise—X

1. State the principle of conservation of energy and give an illustration.

A railway train is moving with uniform speed (*a*) on a leve country, (*b*) up-hill. Explain how the energy supplied by burning coal in the engine is being expended in the two cases. [*C. U.—1911.*

2. It is said that most forms of terrestrial energy, are derived ultimately from the sun. Explain the meaning of the statement, and discuss its truth with special reference to the energy of combustion of charcoal and of coal gas, and the kinetic energy of a running stream. [*C. U.—1912.*

3. State the principle of conservation of energy. Illustrate the principle by taking some simple examples. [*C. U—1913.*

4. Define 'work' and 'energy'. Give simple examples of transformation of energy.

State also the principle of the conservation of energy.
[*C. U.—1916*

5. Distinguish between *work* and *energy*. A body falls under gravity and strikes the ground Explain how the phenomenon supplies an illustration of the transformation of energy.

Does it also illustrate the principle of the conservation of energy ? How ? [*C. U.—1917.*

6. Distinguish between potential and kinetic energy with illustrations.

A railway train is going uphill with a constant velocity. What is the source from which the energy of the train is supplied ?

Describe the various transformation of energy that go in this case. [*C. U.—1918.*

7. Explain clearly the meaning of the terms 'work' and 'energy'. Illustrate your answer by examples.

A body is projected upwards with a velocity of 64 ft. per second Represent graphically its kinetic energy at any height during the upward journey [$g=32$]. [*C. U.—1919.*

8. Explain clearly what is meant by energy of a body. State the *principle of conversation* of energy.

If clouds were one mile above the earth and rain fell, sufficient to cover one square mile at sea level, $\frac{1}{3}$ inch deep, how much work was done in raising the water to the clouds ? [*C. U.—1920.*

9. Define the terms work, force, pressure.

Show that if a piston is moved along a cylinder against a constant pressure, the work done in a stroke is equal to the product of the pressure into the volume swept out by the piston. Explain clearly the units in which the work will be given by this calculation. [*Pat. U.—1921.*

10. Explain clearly what you understand by Work and Energy. State the principle of Conservation of Energy and illustrate it with two examples.

If the clouds were three-fourths of a mile above the earth, and rain enough fell to cover half a square mile of the surface half an inch deep, how much work was done in raising the water to the clouds ? From what did the energy come ?
[*C. U.—1922.*

11. Explain where the energy goes when you expend it in (a) winding up a watch, (b) lifting a stone from the floor and placing it on a shelf, (c) riding a bicycle uphill, (d) rowing a boat on a still pond, (e) rowing a boat up-stream.

[*Pat, U.—1921*

12. Define Work, Energy, and Horse-power, and distinguish between Kinetic and Potential energies

Write a short note on the principle of the Conservation of Energy. [*C. U.—1923*

13 What are kinetic energy, potential energy, and work ?

Find the energy stored in a train weighing 250 tons and travelling at 60 miles per hour How much energy must be added to the train to increase its speed to 65 miles per hour ?

[*C. U.- 1925*

PART—II

PROPERTIES OF MATTER

CHAPTER XI

CONSTITUTION OF MATTER

99. Classification of Matter in Chemistry.—The the various forms of matter with which we are acquainted may be divided into two great classes, viz, *Elements* and *Compounds*.

An **element** is a substance out of which no more than one kind of matter has been obtained. A **Compound** is a substance out of which two or more different kinds of matter (*i e.*, elements having properties *essentially different* from those of the substance) have been obtained. Thus sulphur, iron, mercury hydrogen, oxygen are elements, for each one of these is made up of matter of *one* kind only ; while water consists of the two gaseous elements, hydrogen and oxygen which have properties quite different from those of water. Similarly common salt which is composed of chlorine and sodium, sugar of carbon and hydrogen and oxygen, marble of calcium, carbon and oxygen are all compounds.

Elements and compounds.

The force in virtue of which different kinds of matter unite to form a compound is called the force of *Chemical Attraction* or *Chemical Affinity*.

The elements, the number of which is at present known to be some ninety are divided into two classes, the *metals* and the *non-metals*. The metals, such as Gold, Platinum, Iron, Copper etc, possess a kind of metallic lustre, are generally opaque and good conductors of heat and electricity.

100. Compounds and Mixtures.—Chemical compounds are however to be distinguished from Physical mixtures :—

(1) In a mechanical mixture the properties of the ingredients remain unchanged, while these are entirely changed in a chemical compound.

Expt. 38. Mix together some iron filings and finely divided sulphur. The mixture will have all the properties of iron and sulphur. Present a magnet to the mixture. It will attract the particles of iron leaving the sulphur behind.

Treat a little of the mixture with carbon bisulphide in a test-tube. Sulphur will be dissolved leaving the iron unaltered. Filter the solution, take it in a basin and expose it to the air. Carbon bisulphide will evaporate and crystals of sulphur will be left in the basin.

Heat next a little of the mixture in a test-tube. The mixture fuses and becomes red-hot. A compound of iron and sulphur is formed. This compound is no longer attracted by a magnet; nor is it soluble in carbon bisulphide. The properties of iron and sulphur are changed in forming the compound

(2) In a mixture the ingredients may be present in any proportion by weight. In a compound these are present in definite proportions by weight. Thus in iron sulphide the proportion by weight of iron and sulphur is fixed, viz., in the ratio of seven parts by weight of iron to four of sulphur. The excess, if any, of the ingredient is removable by purely physical means.

(3) In the formation of a chemical compound heat is either evolved or absorbed,—in that of a mechanical mixture heat is neither evolved nor absorbed. Thus in the experiment mentioned above, heat must be applied, so that iron and sulphur may form a chemical compound.

101. Classification of Matter in Physics.—Material substances are found to exist in three distinct stages,—the solid, the liquid and the gaseous.

A **solid** is a form of matter which has a definite shape, volume and mass. It does not require any lateral support to maintain its shape and resists any

change of shape. Wood, stones, ice etc., are solid. The solids are more or less hard.

A **liquid** has a definite volume and mass, but *no shape* of its own. When at rest, it takes the shape of the vessel in which it is contained and maintains a free horizontal surface. Water, oils, alcohol, mercury etc are liquids.

A **gas** has a definite mass but *no shape or volume.* It completely fills the vessel in which it is enclosed, whatever may be the volume of the vessel. Gases are continually tending to expand *i.e.*, to occupy a large space. Hydrogen, oxygen, air, carbon dioxide etc, are gases.

Liquids and gases are comprised under the general name of **fluids.** A Fluid is a substance which flows when unsupported, for its particles are more or less mobile.

It may be found, however, that there are substances which occupy an intermediate stage between the states of the solid and the liquid. For example, sealing-wax and pitch are semi-solids which do not retain their forms when left to themselves. Again liquids such as treacle, honey, syrup, tar etc., do not rapidly assume a free horizontal surface They are said to be *viscous.*

There further is what is known as a *critical state* of matter when a substance may be regarded both as a liquid and as a gas. This is a transitional stage when a liquid is imperceptibly passing into the gaseous state or *vice versa* Again the state of extreme rarefaction of a gas, as in Crooke's tube, is sometimes called the *Fourth State* of matter.

By the application of heat solids may be changed to liquids and liquids to gases. Conversely, the processes may be reversed by lowering the temperature of bodies and increasing the pressure, if necessary. Thus ice when warmed becomes water which may be boiled to generate steam ; steam on cooling changes into water which again exposed to great cold becomes solid in the form of ice.

102. The Constitution of Matter.—Suppose a piece of matter is divided and subdivided into smaller and smaller parts. It is to be admitted that even in imagination the process of subdivision must stop at a certain stage. The ultimate particles into which it can be physically divided and which still continues to retain the properties of matter of the same particular kind are its *molecules.* Thus in the case of a piece of chalk the molecules are the smallest particles of chalk, still possessing all the properties of chalk.

A **Molecule** is the smallest particle of any kind of matter which can exist independently and still preserve the charcter of that kind of matter

Chemical experiments go to indicate that the molecule of a substance is itself divisible into component parts, called *Atoms.* An **Atom** in chemistry is supposed to be the smallest portion of an element that takes part in a chemical combination. A molecule of water is a group of three atoms, one of oxygen and two of hydrogen. Similarly, a molecule of chalk is composed of five atoms, one of calcium, one of carbon and three of oxygen.

Thus a body is an aggregate of molecules, each of which is again supposed to be made up of a group of atoms held together by the inter-atomic forces called the *Chemical Affinity.* In physics the molecule is regarded as the unit in the constitution of matter of a particular kind.

We have no definite idea as regards the '*form*' of a molecule. As to its *size,* it is clear that it is inconceivably small. Approximate calculations have been made by scientists based on different experiments. Thus it has been calculated that a cubic millimetre of water, which is about the size of a pin's head, would contain a number of molecules equal to the cube of a million ; that is a number approximately represented by unity followed by eighteen zeros. Some idea as

Molecules.

to the probable size of a molecule, considered as a small spherical particle, may be obtained from an illustration by LORD KELVIN, in which he states that if a drop of water were magnified to the size of the earth, molecules in it would be about the size of cricket balls !

When pressure is applied to a body or its temperature is lowered, the volume of the body is diminished ; conversely, the length of a rod is increased, when a pull is exerted on it, or its temperature is raised. Again when sugar is dissolved in water, there is no corresponding increase in the volume of the solution. Such facts are best explained by supposing that the molecules which build up a body are not in actual contact. There are *Intermolecular Spaces*, *i. e.*, the spaces between molecules, which are believed to be filled with the *ether*, and which may be altered by the application of external physical forces.

Intermolecular spaces.

The molecules of a body are held together by means of a force, called the *Intermolecular force*. This force is very strong when the distance between the molecules is very small compared to their dimensions, but is very small and almost vanishes when the distance exceeds a certain value. Thus it requires a large force to separate one part of a piece of lead from its adjacent parts ; but if they are separated by even a very small distance, as when the lead is cut by a knife, the two parts will not hold each other even when they are pressed together by a very great pressure.

Intermolecular forces.

To counteract the effect of these attractive forces which, had they existed alone, would have pulled the molecules into the closest contact possible, so that no pressure could have compressed them closer, it is assumed that the molecules of a body are not at rest, but in a state of *rapid motion*, due to which there is a tendency of the body to expand.

Molecular Energy.

Thus the molecules in a body possess the *molecular potential energy* which is due to their configuration in the building up of the body and *molecular kinetic energy* in virtue of their motion (see art. 95.)

The physical state of a body at a given temperature is the result of the balance between two opposing tendencies: one, a tendency to become as dense as possible due to the attractive action of the intermolecular forces, and the other,—a tendency to expand in virtue of the motion of the molecules.

Solid.

In the *solid state* the intermolecular forces are very powerful, so that the molecules of a solid body cling together with great force. This enables the solid to have a definite *form and volume*. It is due to these forces that a solid offers resistance to any change in shape or volume. Thus it requires an enormous force, about 9 tons, to break a steel rod of 1 cm in diameter into two pieces. Yet the molecules in a solid are supposed to vibrate rapidly about practically the same positions with reference to the others (art. 118.)

Liquid.

In the case of a liquid the molecular attraction is rather feeble but still perceptible. A liquid has, therefore, no definite shape, but takes the shape of the vessel in which it is contained. It yields to a force, however small, which tends to change the shape of the mass, and flows out in a direction in which it is free to move. It gives rise to the phenomena which are called the *Surface Tension* phenomena. The properties of a liquid are more fully considered in Chap. XIV.

Gas

The molecules in a gas are supposed to be so far apart from one another that the intermolecular forces are negligibly small or practically absent. The molecules tend to go away in any direction independently of each other. In accordance with the theory, known as the *Kinetic*

Theory of Gases, it is supposed that the molecules of a gaseous body are moving about bodily with great velocity and that they are colliding with each other and bombarding the walls of the vessel in which they are enclosed. This latter causes the *pressure* which the gas exerts upon the surface enclosing it. So all the molecular energy in a gas is of the form of the kinetic energy.

The Electrons.

Recent experiments in electricity have convinced the scientists that an atom is not the ultimate indivisible particle of matter. According to the *Electron Theory*, as has been formulated by Sir J. J. Thomson, and advanced by Sir Earnest Rutherford Prof. Neil Bohr, Sir Oliver Lodge, Dr. Soddy and others the atom is supposed to be made up of two kinds of electrically charged organised ether,—called the **Protons** and **Electrons** : the atoms of different elements differ only in the number of these particles which go to build them up. Dr. Rutherford advanced a theory that it seems probable that the atoms of one substance may change into simpler atoms of another substance by a process of gradual disintegration. The above has gained support from the discovery by Sir William Ramsay and Dr. Soddy (in 1903) that the element Helium seems to be continually produced from the element Radium as a result of this disintegration. Experiments performed in 1905 suggested rather strongly that Radium is being derived from the disintegration of the element Uranium ! It is left to the scientists to declare whether the root of the diverse substances in the world is the one and the same element !

Exercise.—XI.

1. Explain the following terms :—

   Molecule, atom, ether, molecular kinetic energy.

2. Write a short note on the structure of bodies.

CHAPTER XII

GENERAL PROPERTIES OF MATTER.

103. Properties of Matter—We have remarked in art. 2 that the matter is the stuff or the material of which bodies are composed, and which is perceptible by us through our senses. The **Mass** of a body is the total quantity of matter contained in the body. Matter is best defined, however, by the enumeration of its essential properties.

Properties that are found in common in all the states of matter, whether solid, liquid or gaseous, may be called the **General Properties** of matter, *e. g.*, extension, impenetrability, intertia, gravitation, divisibility, porosity, elasticity and density.

Special Properties are such as are found in particular states of bodies. Thus there are special properties of solids, of liquids and of gases. These will be taken up later on.

104. Extension.—Every piece of matter must occupy some definite volume of space. This property of matter is called *extension* or *magnitude*.

Extension regarded in one direction gives a *length*; in two directions, a *surface*; in three directions, a *volume*.

105. Impenetrability.—Two pieces of matter can not occupy the same space. **Impenetrability** is the property of matter in virtue of which a piece of matter occupies a space to the exclusion of all others. If a block of iron is immersed in water, the liquid moves away to make room for the immersed body.

Expt. 39. Put a small bottle in water with its mouth downwards. It will not be filled with water till the mouth is turned, so that the air in the bottle can escape.

The above simple experiment proves that the air is impenetrable.

The term impenetrability is, however, not to be taken in the ordinary sense that one body cannot penetrate another. A nail can be driven into a block of wood. Water poured upon a heap of sand disappears quickly. If 50 c. c. of alcohol is mixed with 50 c. c. of water, the volume of the mixture is less than 100 c. c. The true explanation, in all these cases is afforded by the existence of intermolecular spaces within the body (see art. 102). Thus the molecules of wood are thrust aside to make a space large enough to admit the substance of the nail. In the case of water disappearing in sand, the water does not penetrate the substance of the sand itself, but simply fills the space between the grains.

In fact, impenetrability and extension are not two different properties of matter independent of each other, but are merely two expressions for one and the same thing.

106. Inertia.—Matter cannot, of itself, change its state of rest or motion. Inertia is a characteristic property of matter and is defined in Newton's first law of motion (see art. 36). It is the property in virtue of which a body *i. e*, every form of matter continues in its state of rest or uniform motion in a straight line, unless it is acted upon by an external force to change that state.

There is little doubt in accepting the fact that a body at rest would, if left to itself, continue to be at rest. It is not, however, so obvious in the case of a body in motion that it, left to itself, would continue to move forever with uniform velocity in a straight line, for we are accustomed to see bodies gradually move more slowly and ultimately stops. The true explanation, here, is that it is impossible in practice to free a body entirely from the action of external forces which always impede motion in a greater or less degree. General observation shows, how-

ever, that the more completely a body is isolated from the action of a force, the less appreciable becomes the change of its state of motion.

Thus a marble ball set rolling over the ordinary surface of ground soon comes to stop. The forces of friction of the ground and the resisting force of the air oppose the motion of the ball, and cause it to stop finally by gradually reducing the motion. With a smooth surface as that of a sheet of ice the ball will roll along for a considerable distance before it stops. If all the impeding causes, such as friction against the supports and the resistance of the air, could be removed, a body once in motion would continue to move for ever. In the case of heavenly bodies only such conditions are to be met with.

In virtue of inertia a body tends to oppose any change of its *state*, whether of motion or of rest.

Inertia of motion.

If a train or carriage of any kind comes to stop suddenly, the passengers and the things within, unless they are securely fixed in position, tend to continue in their motion even after the carriage has stopped. Thus they are apparently thrown forward against whatever may be in front of them.

Similar is the experience of a person who steps out carelessly of a moving car ; while his feet on touching the ground are brought to rest, the upper parts of his body still continue to move on by virtue of inertia, in the direction of motion of the carriage with the result that he falls forwards. Same is the explanation when a man in running strikes his foot against an obstacle and tends to fall forwards.

Again, we know from experience that a ball thrown vertically upwards from the hand inside a carriage in motion returns to the hand. While in the air, its horizontal motion which is the same as that of the train at the instant of leaving the hand is practically unaffected. Hence if the motion of the train does not change, the ball keeps

vertically over the hand while in the air and ultimately returns to the hand.

In a circus, when a man standing on a horse which is going at a constant speed is to jump through a hoop, he springs up in a vertical direction. As along with his vertical motion he also continues to keep his horizontal velocity which is the same as that of the horse, he falls again on the back of the horse after passing through the hoop.

Before taking a long jump a person often runs from a little distance off, so that the motion of his body at the time of taking the leap may add to the effort then exerted.

Instances of inertia of rest are equally numerous. When a horse at rest suddenly gallops away, the rider is thrown backwards in his seat due to the tendency of the upper parts of his body to continue still in the state of rest.

Inertia of rest

Expt. 40. Place a coin on a card (fig. 103) covering a cup or the mouth of a wide-necked bottle. Strike the card suddenly in its own plane so that it flies away. The motion of the card is so rapid that the coin possessing inertia of rest has little time to share it, and being unsupported when the card flies away, drops into the bottle.

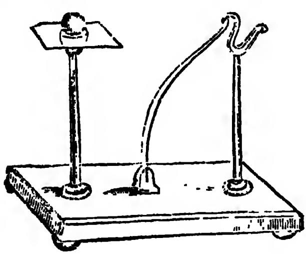


FIG. 103.

In fig. 103 the bent spring when released from the catch makes the card fly.

Thus a bullet fired against a window pane will make a clean hole in it ; but with a less violent blow as that of a stone thrown against it, the pane will be smashed.

The action of beating a coat with a stick to expel dust, of cleaning a dusty book by striking it against another, the flying off of mud tangentially from a rotating carriage wheel do all depend on inertia.

107. Gravitation.—Every piece of matter in the universe has the power to attract every other piece of matter; this is a fundamental property of matter. The force of attraction exerted mutually between any two pieces of matter is called *Gravitation.*

The amount of this attraction depends on the quantity of matter in each and their distance from each other; it is greater when bodies have large masses than when they have small ones, and also when they are nearer than when they are more remote

It is this force of gravitation that acts as an invisible cord in restraining the moon in her path round the earth, and the earth and other planets in their respective orbits round the sun. Owing to this the earth draws all bodies towards it. It is gravitation that accounts for the daily occurrence of tides in the ocean; in full moon and new moon, the combined action of the sun and the moon produces spring-tides, while in the first and last quarters the difference between their action causes neap-tides.

The attraction between the earth and bodies on it is, as has already been mentioned, known as Gravity and is the cause of their weight. The enormity of the size of the earth makes the earth's pull on a body on or near to its surface so great, that the effect of any other small mass on it is quite masked. Hence two balls hanging by two strings do not ordinarily seem to approach but careful experiments with very delicate means of observation have been able to detect a slight approach towards each other.*

The discovery of the Laws of gravitation is one of the greatest discoveries in Newton's life (1642-1727). Newton was born in 1642 (the same year in which Galileo died) at Woolsthrope, near Grantham in Lincolnshire. In 1666 while he was at Cambridge, he first thought out the great theory of Gravitation. In the course of his astronomical studies, Newton had come across a problem which he could not solve.

Newton's discovery of the Laws of Gravitation.

* See Poynting and Thomson's *Properties of Matter.*

The problem was this :—Why does the moon always move round the earth and the planets round the sun ? The natural thing is for a body to go straight on ; *why then, should the bodies in the sky go round and round, and not straight forward* ?

"While Newton was still pondering over this question, the plague broke out in Cambridge in the year 1665, and he was forced to go back to Woolsthrope. Here he was sitting one day in the garden meditating as usual, when an apple from the tree before him snapped from its stalk and fell to the ground This simple incident attracted Newton's attention ; he asked himself, '*why does the apple fall* ?' And the answer he found was '*Because the earth pulls it.*'

This was not quite a new thought, for many clever men before Newton had imagined that things were held down to the earth by a kind of force, but they had never made any use of the idea. Newton, on the contrary, seized upon it at once, and began to reason farther. "If the earth pulls the apple," argued he, "and not only the apple but things very high up in the air, should it not pull the moon, and so keep it going round the earth instead of moving on in a straight line ? And if the earth pulls the moon, may not the sun in the same way pull the earth and the planets and so keep them going round and round with the sun as their centre, just as if they were all held to it by invisible strings ?" —*Buckley's Short History of Natural Science*.

Newton felt convinced that the guess was all right but he set to work to prove it by very difficult calculations ; for example, what the effect would be on the motion of the moon, if it was true that the latter was attracted by the earth. For *sixteen years* he had to wait patiently before he worked out his hypothesis to his satisfaction : it was in 1666, when he was only twenty-four, that he saw the apple fall, and it was in 1682 that he gave out to the world his famous Theory of Gravitation.

Owing to the feebleness of attraction between bodies that can be handled, the real existence of the mutual attraction between bodies could not be directly demonstrated ; but astronomical calculations involving the forces acting between two bodies moving in space, and made according to these laws, yield most accurate results. Thus the motion of the planets round the sun can be satisfactorily explained, formation of tides upon the waters of the earth under the influence of the Moon and the Sun is understood, and eclipses are successfully predicted.

The *Laws of Gravitation* may be stated as follows :—

(*a*) Every material body in nature attracts every other material body towards itself; in other words, any two material bodies in nature are exerting equal forces of attraction on each other in opposite directions. along the line joining the two.

(*b*) The force of attraction between two particles of matter is *directly* proportional to the product of their masses, and *inversely* proportional to the square of the distance between them.

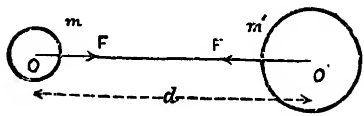


FIG. 104.
Force of Gravitation.

Thus if m and m' in fig. 104 represent the masses of two *particles* placed at a distance d apart, the force of attraction which each exerts on the other is such that we have

$$F \propto \frac{m.\, m'}{d^2}$$

$$\text{or} \qquad F = G.\, \frac{m.\, m'}{d^2}$$

where G is a constant, known as the *Gravitational Constant* which is independent of the nature of the masses.

When $m=m'=1$ and $d=1$, then $G=F$ and represent the force which two bodies, each of unit mass, exert on each other, when they are unit distance apart. The value of G has been determined by CAVENDISH and others. As given by BOYS, its value is $6{\cdot}6576 \times 10^{-8}$ in C. G. S. units which is a very small quantity.

The distance d in the above formula is strictly the distance between two *particles* of matter. In the case of spheres of uniform density it is the distance between their centres, so that the bodies attract each other as if their masses were concentrated at their centres.

Thus it follows from the laws that the mutual attractions between two spheres are equal to each other, and each equals $G. \frac{m. m'}{d^2}$. If the mass of either sphere be doubled, trebled etc, the force of attraction will also be doubled, trebled etc. Again, if masses remaining the same, the distance between their centres be doubled, trebled, etc, the force of attraction will be diminished to $\frac{1}{4}$, $\frac{1}{9}$ etc. of the original force and so on.

Gravity.

If in the above formula, we take one of the masses as M, the mass of the earth, and the other m, as that of a small body at the surface of the earth and R, the radius of the earth, we have

$$F = G. \frac{M. m.}{R^2}$$

where F is the force of gravitation exerted by the earth on a terrestrial body and is for convenience given the special name of **Gravity.** So gravity is only a particular case of the universal gravitation.

Thus not only the earth of mass M attracts a rain-drop of mass m, but the drop also attracts the earth. As action and reaction are equal and opposite, the value of the attractive forces is $G. \frac{M.m}{d^2}$. The force imparts to the rain-drop an acceleration which equals $G \frac{M}{d^2}$ and to the earth the acceleration $G. \frac{m}{d^2}$; but as the mass of the earth is many millions of times larger than that of the drop, the earth seems, for all practical purposes, to be at rest, while the rain-drop falls on the earth.

108. Divisibility.—It is the property, in virtue of which a body can be divided into extremely small parts.

Bodies can be subdivided into smaller parts by mechanical methods, such as cutting, sawing, filing grinding etc. A piece of chalk is divided into a large number of detached particles when one writes on a

board with it. Gold can be hammered into leaves so thin that 300,000 of them make 1 inch thick. Glass, platinum, quartz have been drawn into fibres, so fine as to be quite invisible.

Very fine divisions are frequently effected by means of solution. A drop of carmine will colour a litre of water perceptibly red. It is calculated that a drop of blood which may be held on the point of a needle would contain about a million of blood-corpuscles floating in a colourless liquid, called the serum.

Expt. 41. Put a small crystal of potassium permanganate or a drop of ink in a beaker of water. The extreme minuteness of the particles is well shown by the spreading of the colouring matter.

Expt. 42 Put a particle of common salt on the clean wick of a spirit-lamp. Note that the sodium vapour gives the flame a yellow colour for hours

Still greater is the divisibility of oderiferous substances such as volatile essences, camphor, musk etc. The tenth part of a grain of musk is found to perfume a large room for years together and yet lose but little of its weight.

There is, however, a limit to the divisibility of matter. In the process of subdivision there must come a stage when further subdivision will break up the ultimate particle, called the *molecule* of the given kind of matter into its constituent components, which are called atoms in chemistry (see art 100)

109. Porosity—In a body the very small parts are not in actual contact but they leave spaces or interstices between them. This is expressed by saying that bodies are porous.

Expt. 43. Take a lump of dry chalk and weigh it carefully. Next place the chalk in water for a few minutes. Air bubbles will be seen to escape from the pores of the chalk, being displaced by the entering water particles. Now weigh the wet chalk and notice that its weight has increased.

Expt. 44. Take a tumbler brimful of warm water. Dissolve some sugar in it. It will be seen that a large amount may be dissolved without any overflowing of water from the tumbler.

Pores are to be distinguished from the intermolecular spaces. The latter are never directly perceptible and so very small that the surrounding molecules remain within the sphere of each other's attraction. Contractions and expansions of bodies resulting from changes of temperature are explained by the existance of these intermolecular spaces. *Pores* are all actual cavities or holes which are sometimes distinctly visible under the microscope. They are too large for the intermolecular forces to act.

Fine pores exist in wood, cork, sponge, unglazed earthenware, paper, leather, india-rubber, pitch, cloth, skin etc. The existence of pores in leather is demonstrated in the following experiment, known as the *Mercury Rain.*

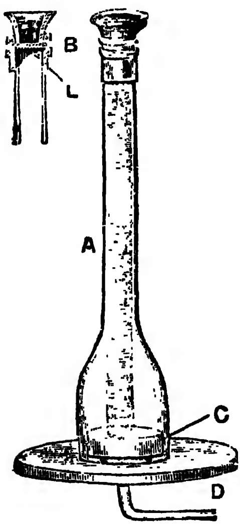


FIG. 105.
Mercury Rain.

Expt. 45. A long glass tube A (fig. 105) is provided with a brass cup B at the top. The bottom of the cup consists of a thick piece of leather L. The lower part of A is wide enough to admit a cup C to be placed within it.

Place A on the disc D and apply a little grease all round the base of A. Pour mercury to partially fill up the cavity B at the top. Work the pump to reduce the pressure of air within the tube. A shower of mercury is produced within the tube, as mercury is forced through the pores in the leather by the greater pressure of atmosphere outside. The vessel C is used so that mercury may not fall on the pump disc.

In filtration a liquid is freed of the particles suspended in it by allowing it to percolate through charcoal or carbon. The chemists use the filter-paper for this purpose in the laboratory. The blotting-paper is used for soaking ink: powders of chalk

may also be used for this purpose. Deep well water is generally clear as it is filtered on its way through thick strata of the earth. It is to be noted that a filter is of no use to keep back *dissolved* impurities; for example, salt-water when passed through a filter retains its saline taste.

The inter-penetration of two liquids, such as alcohol and water, is often chosen as an example of porosity in liquids.

110. **Elasticity**—Elasticity is the property in virtue of which a body is able to resist a change of volume or of shape, and is able to resume its original volume or shape when the force that caused the change is removed, provided that the change has not exceeded a certain limit which, of course, differs for different materials.

Expt. 46. Squeeze a piece of india-rubber. It returns to its original shape as soon as the force is withdrawn from it.

When a force or system of forces act on a body which is not free to rotate, or to move bodily, a displacement of the particles relatively to one another is caused. Such a change in the shape or volume of the body is called a **Strain.**

In general, when a body is strained by the application of external forces, it resists the strain by setting up forces within the material, which not only oppose the change, but tend to restore the displaced particles to their original positions. The restoring force generated in the body due to the strain is called a **Stress.**

A body is able to offer this elastic resistance, when the strain is small and does not exceed a certain limit called the *Elastic Limit* for that material. Since within the elastic limits the stress generated within the body is equal and opposite to the external force producing the strain, the latter also is often called the stress. Hence we say that a stress applied to a body produces a strain.

Hooke established experimentally that if a body is strained *within its elastic limits*, the strain produced is directly proportional to the stress applied,

$$i.e., \frac{\text{Stress}}{\text{Strain}} = \text{a constant},$$

which is called the **Modulus of Elasticity** of the material of the body for the particular kind of stress and strain under consideration. *The modulus* is defined as *the stress that is required to produce an unit strain.* The above relation is known as **Hooke's Law.**

Kinds of strain.

Strains produced by applied forces may be divided into two classes, *according as they consist of a change in volume only or a change in shape only*. The former is called the *Volume Strain*; while the latter, the *Shearing Strain* or simply a *Shear* (see art. 116).

Elasticity of volume is possessed by bodies in all the three states of matter The modulus concerned can be expressed in symbols as follows :—

If a uniform pressure of p dynes per sq. cm., acting normally to the surface of a substance, be applied to reduced the volume V to $V-v$, then

the stress $= p$ dynes per unit area,

and the vol. strain $=$ deformation produced per unit volume

$= v/V$

$$\therefore \quad \text{The modulus of vol. elasticity} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{p}{v/V}$$

Illustrations of the elasticity in solids are numerous. Thus corks pressed into the necks of bottles fit tightly. Elasticity of horse-hair, feather, wool, cocoanut fibres etc., is made use of in pillows, seats, cushions etc.

A liquid offers a very great resistance to a change in size, but almost no resistance to any change in shape; in other words, a liquid possesses a high modulus of elasticity of volume, but is devoid of elasticity of

shape. This forms an important difference between a solid and a liquid.

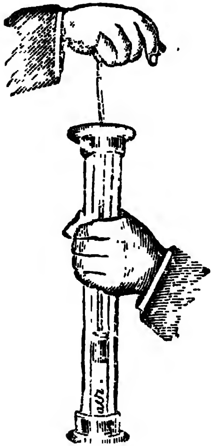


Fig. 106.

A gas, like a liquid, has elasticity of volume but no elasticity of shape.

Expt. 47. Fig. 106 represents a stout glass tube closed at one end and provided with a tightly fitting piston. Press the piston down to compress the enclosed air Note that a gas is a most compressible body. Observe also that as soon as the pressure is removed, the piston is pushed up to its initial position.

111. Density.—If a body be homogeneous if *i.e.*, the matter contained in it be uniformly distributed through it, the mass per unit volume for the body is constant, and hence characteristic of that substance, and is called the **Density** of the substance. Thus, in the case of a homogerous body, if we have

M = mass of a body of uniform material.

v = volume of the body,

ρ = density of the material,

then $\rho = \frac{M}{v}$ or $M = v\rho$

When the body is not homogenous, the density at any point of it is obtained by taking a unit volume round this point, and finding the mass contained in that unit volume, the supposition being that the ditribution of matter through such a small volume is uniform.

Different substances may contain very different quantites of matter in the same volume. Thus the mass of 1 c. c, of pure water at 4°C is 1 gramme, that of

1 c. c. of copper is about 8·9 gms, that of silver 10·5 gms, that of lead about 11.3 gms, that of gold about 19 gms. and so on.

Density as expressed above is sometimes called the *absolute density*. But the density of a substance may also be expressed *relatively* with reference to the density of some substance taken as the standard. The standard substance chosen for this purpose is pure water at 4°C. Thus the relative density of a substance is a ratio of the density of the substance to that of water at 4°C and is merely a number.

Relative density.

Thus the absolute density of lead is 11.3 gms. per cub. cm. and that of water at 4°C is 1 gm c. c., so that the relative density of lead is 11·3.

In the C. G. S. system the relative density and the absolute density are both expressed by the same number. But there is no such advantage in the F. P. S. system. For example, the absolute density of lead in the English system of units is about 706 pounds per cu. ft. And that of water is about 62·5 lbs. per cu. ft., so that the relative density of lead is $\frac{706}{62 \cdot 5}$ or about 11·3. It should be noticed that whatever may be the units in which the absolute density is measured, the relative density will be expressed by the same number in every system of units.

The **Specific Gravity** of a substance is defined as the *ratio* of the weight of any volume of the substance to that the same volume of water at 4°C. Hence it is a pure number.

Thus if W' denote the weight of a volume V of a substance and W, the weight of the same volume of water at 4°C, then

$$\text{Sp. Gr.} = \frac{W'}{W} = \frac{M'g}{Mg} = \frac{M'}{M} = \frac{v\rho'}{v\rho} = \frac{\rho'}{\rho}$$

$$= \frac{\text{density of the substance}}{\text{density of water at } 4\,\text{C}}$$

Hence specific gravity is the same as the relative density of the substance.

Change of density with temperature.

In general, the volume of a body increases with the rise of temperature, hence for a given mass the density must decrease as the temperature rises. An important exception to this rule occurs in the case of water, when its temperature lies between 0°C and 4°C Within these two temperatures water contracts when heated and expands when cooled. At 4°C., it attains its maximum density. Above 4°C., it behaves in the ordinary way *viz*, it expands when heated. Fig. 107 given below represents graphically

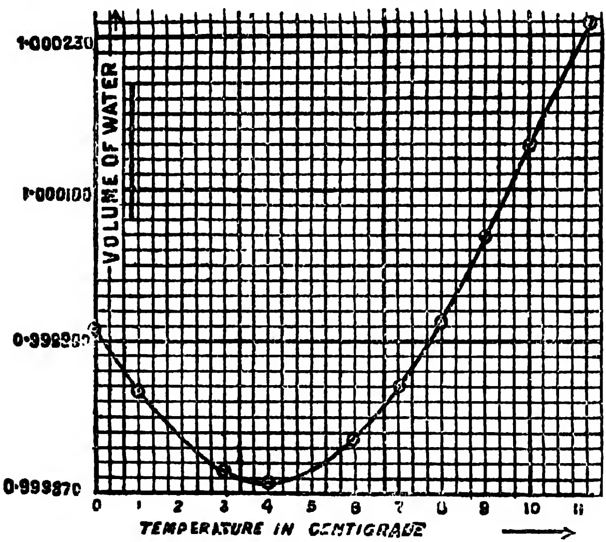


FIG. 107.
Graph to represent the maximum density of water at 4°C.

the changes in volume and hence the changes in density of water with temperature.

A table of the relative densities of a few of the most common solids and liquids is given on the next page.

The density of gases, expressed with reference to that of hydrogen, is considered later on. Thus the absolute density of dry air at 0°C and under normal atmospheric pressure *i. e.*, at N. T. P., is 1.293 gms per litre, or ·001293 gms. per c c.

Table of Relative Densities or Specific Gravities.

Solids.

Aluminium ...	2·6	Iron (wrought) ...	7·86
Ash ...	·9	Ivory ...	1·9
Brass .	8·5	Lead ...	11·3
Brick ...	2·1	Marble ...	2·7
Carbon (graphite) ..	2 1	Nickel ...	8·6
Chalk ...	1·9-2 8	Paraffin wax ..	0 9
Coal ..	1·2-1 8	Platinum ..	21·5
Copper .	8·9	Porcelain ..	2·5
Cork ..	0·24	Sand ...	1·6
Ebonite ..	1·1	Silver ...	10·5
German silver ...	8 4	Slate ...	2·6
Glass ...	2·6	Sugar ..	1·6
Gold .	19 3	Sulphur ...	2·05
India-rubber .	·9	Tin ...	7·3
Iron (cast) ..	7·4	Zinc ...	7·1

Liquids

Alcohol ..	0·79	Nitric acid ...	1·6
Carbon Bisulphide ..	1·29	Sea water ..	1·02
Glycerine .	1·26	Sulphuric acid ...	1·85
Hydrochloric acid	1·27	Water at 4°C ...	1·0
Mercury ...	13·6		

Exercise.—XII

1. Enumerate the general properties of matter. Explain Inertia, citing illustrations.

2. A capillary glass tube weighs 3 gms. A thread of mercury 8 cm. long is drawn into the tube when it is found to weigh ·6 gms. Find the diameter of the capillary tube.

3. Find the volume of a block of lead weighing 100 gms.

4. A rectangular block of wood 5 by 4 by 2 c. cm. weighs 200 gms. Find the density of wood.

5. Give a examples to illustrate

(*a*) the extreme divisibility of matter :
(*b*) gravitation in the case of heavenly bodies :
(*c*) elasticity of tension ;
(*d*) existence of pores in pieces of chalk.

6. State what will happen in the following cases and account for them :—

(*a*) A bullet is fired against a window pane :
(*b*) A much used carpet is beaten with a stick :
(*c*) A tuning fork held by the steam is struck on the table :
(*d*) A person alights from a tram-car in motion,

7. Distinguish between

(*a*) Gravitation and gravity ;
(*b*) Elasticity and compressibility of a quantity of gas.

8. Define specific gravity of a body.

Distinguish between Density and Specific Gravity of a body.
[*C. U.—1921* ; '22.

CHAPTER XIII.

PROPERTIFS OF SOLIDS.

112. Hardness.—When one body can be made to scratch another and can not be scratched by it, the former is said to be *harder* than the latter. Thus glass is harder than lead. Hardness is a relative property; thus a body which is hard in reference to one body may be soft in reference to others. Diamond is the hardest of all substances for it scratches all, but is scratched by none. Hence it can only be ground by means of its own powders.

The degree of hardness of a body is expressed by referring to a scale of hardness. The one generally employed is *Mohr's scale of hardness*, and is as follows, in which the substances are arranged in the order of increasing hardness :—

SCALE OF HARDNESS.

1.	Talc.	6.	Felspar.
2.	Rock-salt.	7.	Quartz.
3.	Calc-spar.	8.	Topaz.
4.	Flour-spar.	9.	Sapphire.
5.	Appatite.	10.	Diamond

Thus the hardness of a body which would scratch felspar but be scratched by quartz lies between 6 and 7.

The pure metals are softer than their alloys. Hence for jewellary and coinage, gold and silver are alloyed with copper to increase their hardness.

113. Brittleness.—This is the property due to which a body breaks down easily under a blow from a hammer. Ice, glass are very brittle substances ; lead, copper are not.

By heating and sudden cooling many substances especially steel, become very hard though they become brittle at the same time The process is called *Tempering*. By re-heating and slow cooling, which is called *annealing*, steel, glass etc., can be made less brittle.

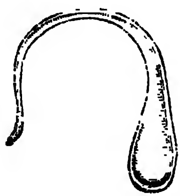


FIG. 108.
Rupert's Drop.

In Rupert's Drops (fig. 108) which are formed by dropping melted glass in cold water the glass is in a state of brittleness. When the fine point of any one of these is just broken off, the whole mass falls at once into fine powder.

114. Malleability.—It is the property possessed by some solids of being beaten into thin sheets. Pure gold is exetremely malleable. Leaves of gold are obtained which are so thin that 300,000 of these, when superposed, become only an inch thick. Tin-foil is an alloy of lead and tin beaten thin ; tin-plate or block-tin is sheet iron covered with tin. Platinum-foil is used for electric batteries.

Malleability increases quickly with the rise of temperature, thus iron is easily forged when hot but not when cold.

115. Ductility—It is the property of solids of being drawn into fine wires.

Platinum is the most ductile of all metals. It can be drawn out into wires so thin that 140 of such wires placed, side by side, have the thickness of a silk-fibre. Gold, silver, copper, iron etc. can also be drawn into wires by means of a machine, called the *Draw-plate* in which the rod of a metal is

successively pulled through a number of holes, each a little smaller than the last, bored in a plate of steel. Lead which is very malleable, is the least ductile.

Extremely fine wires of glass, quartz etc, can be drawn when these are softened by the application of heat.

***116. Elasticity of Shape or Rigidity.**—A solid possesses both the elasticity of volume and the elasticity of shape.

When a solid is strained in such a way that it suffers change in volume only without a change in shape *e g*, when a sphere is strained into another sphere, the elasticity concerned is known as **Volume Elasticity** or **Bulk Elasticity.** It is seen in art. 110, that the modulus of volume elasticity which is generally denoted by k, is given by

$$k = \frac{p}{v/V} = \frac{pV}{v}.$$

To strain a body in the above way is rather difficult. The applied stress must be a uniform pressure acting normally all over the surface of the body. Pressure may be applied in this way by immersing the body in a suitable liquid and then applying pressure to the liquid, for the pressure on a liquid is transmitted equally in all directions and acts everywhere at right angles to the surface. (See art 130).

The *elasticity of shape* or as is more generally called the **Simple Rigidity** is the constant brought into play, when a solid is so strained as to suffer a change of shape or form without a change of volume. When a cylinder is subjected to a twist or torsion round its axis about one end which is fixed, a shearing strain is produced.

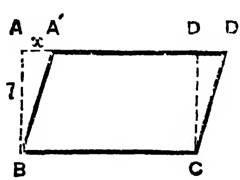


Fig. 109.
Shear Strain.

If a portion of a substance be taken as included in a rectangular block ABCD (fig. 109) and if a stress P per unit area is allowed to act uniformly and tangentially over

the face AD, so as to displace the plane relatively to a fixed parallel plane BC at a distance l from it, the material between the layers is subject to a simple change in form without a change of volume. The strain produced is known as **shearing strain** or a **simple shear.** The modulus of shear, which is generaly generally denoted by μ, is given by

$$\mu = \frac{\text{stress}}{\text{strain}} = \frac{P}{x/l}$$

The changes in the shape of a solid may be done by flexture (or bending), by torsion (or twisting) and by tension (or stretching).

The *Elasticity of Tension* is well seen in india-rubber. If a common india-rubber band is stretched and then released, it contracts to its original length.

Expt. 48. Take an india-rubber band about 50 *cms.* long and attach it to a suitable stand. Hang a scale-pan from the lower end of the band and thrust two small pins P_1, P_2 through the india-rubber at a distance of about 40 *cm.* from each other. Any change in the position of P_1 and P_2 may be read off from the two millimetre scales S_1 and S_2 etched on mirror glass and fixed to the stand (Fig. 110).

Place a load of 50 gms on the pan and, gradually increase it by repeated addition of 50 gms. Determine each time the distance between the two pins. Also take out the weight after each trial and note whether the scale-pan returns exactly to its original position. It will be found that it does so if the stretching weight does not exceed a certain limit.

The strings of musical instruments are kept tightly stretched, so that they vibrate owing to their elasticity of tension when they are distubred in any way, *e.g.*, by bowing as in a violin, by plucking with finger as in a harp, or by striking with padded hammers as in a piano.

In such cases where a wire or rod fixed at one end is stretched by a tension in the direction of its length, the strain produced involves a change both of volume and of form. The modulus of elasticity in this case involving both the volume elasticity and the simple rigidity is given the name of **Young's Modulus** (Y).

If P be the force which acting on a wire of length L

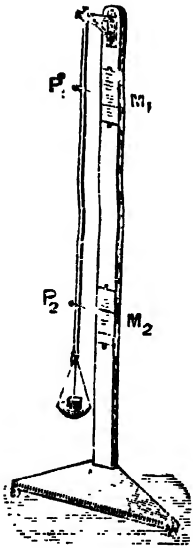


FIG. 110.
Elasticity of an india-rubber band.

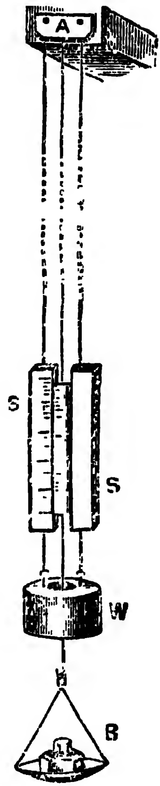


FIG. 111.
Determination of Young's modulus.

and of cross-section a stretches it by a short length l, then

the strain = elongation per unit length = l/L
and stress = force per unit area

$$\frac{P}{} = \frac{P}{}$$

if the cross-section is circular and of radius r. By Hooke's law the modulus concerned is given by

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{P/\pi r^2}{l/L}$$

The value of Y for the material of the wire can be experimentally determined by means of an arrangement shown in fig. 1.1. AB, the wire to be stretched, is fastened to a clamp above and carries a vernier V and a scale-pan B as shown in the figure. Two wires supported from the same clamp A carry two brass pieces S, and S, one of which is divided into a fine scale. The two side-wires are kept stretched by a tubular weight W of lead.

Expt. 49. Place weights on the pan so as to keep the wire taut and free from kinks. Measure the length of the wire AB between the clamp and the vernier by means of a metre-scale.

Measure next the diameter of the wire by taking the mean of several measurements taken with a screw-gauge. Hence find the cross-section.

Place a weight on the pan and gradually add weights to these noting the vernier reading everytime. Hence find the elongation.

To calculate Y apply the formula

$$Y = \frac{P}{\pi r^2} \cdot \frac{L}{l}$$

By subjecting wires of different materials to the above treatment the elastic constants, can be compared, for the other things being constant P varies as Y. Thus it is found that for the same elongation if a steel wire requires 10 kilograms, exactly similar wire of copper requires 6 kilos, that of brass 4·5 kilos and that of platinum 8 kilos, Steel is, therefore, about 1·7 times as elastic as copper. Platinum is nearly twice as elastic as brass.

In the above experiment it will be seen that unless the wire is stretched beyond the limit of elasticity for the material, it reverts to its original length. But even within the elastic limit, when the applied force producing a

Elastic limit.

strain is allowed to act for a long time, the wire does not return to its exact original condition quickly, but takes time to do it : the wire is said to have acquired *elastic fatigue.*

If the load on the wire be gradually increased, a stage is reached when the wire breaks. The weight required for this, is called the **Breaking Weight** and is the measure of the **Tenacity** or tensile strength of the material of the wire The tensile strength of the same material is directly proportional to the cross-section. The breaking stress for a cross-section of 1 sq. mm. is 34 kilos for brass, 30 kilos for copper, 60 kilos for iron and 30 kilos for steel.

Hooke's law is also found to hold in the case of elongation of spiral wires. Hence these may be used for weighting purposes as in spring balances, Jolly's spring etc.

The *Elasticity of Bending* is seen in watch-springs, bows, carriage-springs, spring-balances etc. When an elastic flat piece, such as a steel plate is bent it is obvious that the fibres on the convex side are lengthened and put in tension, while those on the inner side are shortened and compressed. The laws according to which bending takes place in elastic beams, are of considerable interest to the architects. Again, a beam is much less easily bent when laid on its edge than when laid on its face. This is the reason why the beams of the roof of a house are placed on their edges.

Elasticity of steel is utilized in carriage-springs, the buffer springs of railway carriages, clocks and watch springs, spring-balances (fig. 10), spring dumb-bells, dynamo-meters, letter-weights etc

The *Elasticity of Tension or Twist* is brought to action when a wire or a thread is twisted from its original position.

Expt. 50. Take a ball or cylinder and support it from a long, thin wire, the upper end of which is fixed above. Attach

a strip of paper with a little gum at the bottom of the ball to act as a pointer. Rotate the ball and then leave it. The ball returns to its initial position after a few seconds.

This property of torsional elasticity of a wire is utilized in the physical laboratory as a delicate means of measuring magnetic and electric forces *e. g.*, in Torsion-balances, Vibration Magnetometers etc.

Different solids present different degrees of elasticity. Glass, steel, ivory, marble are highly elastic bodies; while substances like lead, clay, fats etc possess scarcely any elasticity, and are called *plastic* substances.

India-rubbber is commonly taken as a highly elastic substance; but the real fact is that it has wide limits of elasticity. It may be stretched to twice or thrice its length, and regain the original length when the force is removed. Glass is much more elastic than india-rubber but its limits are much narrower. The elasticity of solids may roughly be compared in the following way :—

Expt. 51. Smear the surface of a polished marble or slate with a little oil. Drop a glass ball on the slate from a height (fig. 112). On touching the slab it will rebound. At the place of contact the oil will be found to be removed not from a point only but from a small well-defined circle. Repeat the experiment wtih balls of different substances.

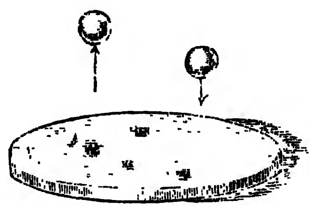


FIG. 112.
Bouncing Balls.

The above experiment shows that at the moment of impact, the ball has a kinetic energy, and has to work through the distance by which the ball is compressed against the opposing forces; in consequence, the ball is flattened at the lower part. After rest, its rebound is caused by an effort of the stresses generated in the ball to recover its original form.

*117. **Cohesion**—From the various physical properties of solids viz., hardness, elasticity, ductility, tenacity *etc.* we are led to suppose that the molecules of a solid cling to one another with forces that are of great magnitude at inconceivably small distances. The attraction exerted by the molecules is called **Cohesion** and the intermolecular forces are called the *Cohesive Forces*. The variation in the physical properties possessed by different solids depends upon the difference in the magnitude of the cohesive forces existing between their molecules.

Molecular forces in solids.

Cohesion is also present in liquids. It acts between molecules of water or of any other liquid and cause them to join and form drops when the liquids are in small quantities. This is seen in rain drops, in dew-drops on the leaves of plants and in small globules of mercury (see also art. 152) In large masses of liquids the force of gravity preponderates over that of cohesion : hence these have no shape of their own, but take the shape of the vessel in which they are contained.

Cohesion in liquids.

Attraction is also exerted between molecules of different kinds : for example, between water and a finger or a glass rod dipped in it To this has been given the name of **Adhesion**. There is no real difference between cohesion and adhesion ; the former is often restricted to the attraction existing between molecules of *like* kinds *i. e.*, the molecular attraction inside a body, while the latter to the attraction be- tween two *different* bodies with their surfaces in contact

Adhesion.

Expt. 52. Pour water, or milk, or any other liquid that wets glass very gently out of a beaker without a spout. Note that it is apt to run down the sides, this being due to adhesion.

Now direct the falling stream by means of a glass rod placed against the edge of the beaker. This time there is no spilling.

If two pieces of plane and brightly polished brass plates or cast-iron plates (A and B in fig 113 be slightly moistened with water and then pressed together to exclude the air, it is found that they adhere so firmly that they can support a considerable weight. As the experiment succeeds in a vaccum by suspending the whole under the receiver of an air-pump, the effect cannot be due to atmospheric pressure.

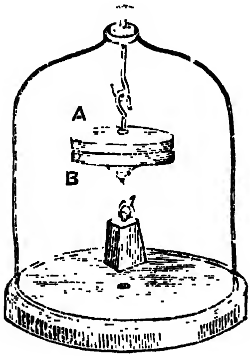


Fig. 113.
Adhesion of surface plates.

Glue sticking to wood, mortar to bricks, nickel plating to iron, copper to zinc in solder *etc*, are instances of adhesion and cohesion ; good adhesion is secured by applying the cement or solder in liquid condition, and cohesion is brought into play when the liquid is dried up. The particles of a crayon are held together by cohesion, while they are held to the blackboard by adhesion.

It is due to adhesion again that two liquids mix with each other *e.g.*, alcohol and water, or that a salt or sugar dissolves in water Oil and water have no adhesion for each other, and hence they do not mix. Resin has no adhension for water and therefore does not dissolve in it. On the other hand heat helps solution, as a rule, because it lessens the cohesion within the soluble substances : for the same reason powdering a solid makes it disolve more readily.

***118. Molecular Motion in Solids**—In the preceding article we have seen that molecules in a solid are acted upon by *cohesive forces*. These forces must be overcome before a change in the shape or volume of solid can be brought about.

To counteract partially the effect of the force of cohesion due to which there is a tendency of the molecules in a solid to come as closed together as possible, it is supposed that such molecules are in a state of rapid oscillation and thereby possess kinetic energy which determines the temperature of the body.

Expansion of a solid when heated.

On applying heat to a body the molecular kinetic energy increases ; the molecules push out to greater distances resulting in the increased volume or expansion of the body. Conversely, by lowering the temperature of a body the velocity and the amplitude of motion of the oscillating molecules are decreased. Thus the iron tyres for carriage wheels are made a little smaller than the wheels which they are to fit. The tyres are heated until they become large enough to be driven on the wheels which are gripped very tightly when the tyres shrink again on cooling.

Elasticity.

At any temperature the volume of a body is a result of the balance between the action of the cohesive force and the kinetic energy of the molecules. When a body is compressed, the molecular motion resists the deformation, if pulled apart, the attractive forces resist the alteration in size ; in other words, the body possesses elasticity.

Sublimation.

The supposition that the molecules of a solid are in motion, which is suggested by the effect of heat on bodies, is strengthened by the fact of *sublimation* of solids. Molecules of iodine pass from the solid to the gaseous condition without passing through the liquid stage at all, when a gentle heat is applied to it. Again if a piece of camphor or musk is kept in one corner of a room, the odour can be detected in the distant corner.

Diffusion of solids.

It has been demonstrated further that if layers of gold and lead are laid on, one upon another, at ordinary temperatures for a great length of time, the molecules of gold are detected

in the mass of the lead. At high temperatures such as 500° C or above, all metals show the same characteristics to a remarkable degree.

The above facts do undoubtedly indicate the existence of molecular motion in solids.

Exercise.—XIII

1. Explain and give illustrations of the different kinds of elasticity possessed by a solid.

2. If a kilogram stretches a wire of 1 *mm.* in diameter ·5 *mm.*, how far will a wire of the same length but of twice the diameter be stretched by 8 kilograms?

CHAPTER XIV

LIQUIDS

119. Hydrostatics.—It is that branch of Physics which deals with the equilibrium of liquids and gases and with the pressure they exert, whether within their own masses or on the sides of the vessels in which they are contained.

120. Special Characteristics of Liquids—It has been noted before in art. 102 that the particles of a fluid are mobile, while those of a solid cling to each other with a strong force.

The essential difference between the behaviour of a solid and that of a fluid is explained below :—

Let a solid body rest in the position of equilibrium. Suppose that a plane, inclined to the horizon, is drawn through the solid dividing it into two parts. The weight of the upper portion would cause it to slide down the plane, had there been no shearing stress, *i.e.*, the force in the nature of friction along the imaginary plane to oppose such motion. Again an attempt to cut a solid through with a knife meets with a great resistance from it. It may be said therefore that *every solid can resist a shearing stress without giving way to an indefinite extent.*

But if a very thin lamina be pushed *edge-ways* through a liquid, say water, or mercury or any oil, the resistance to its motion is very small. If a large quantity of liquid be placed on a flat table, it will spread itself out continuously, for any horizontal layer of the liquid

is subject to a pressure due to the weight of the overlying liquid and as the molecules of the layer are free to move about, the layer is not in the least compressed, but is compelled to flow out horizontally in all directions. When a mass of liquid is at rest in any containing vessel, this outward flow is, however, prevented by the pressure exerted inwards by the walls of the vessel on the edge of the layer.

Thus we may say that a liquid can be ***poured*** from one vessel to another and that when poured in a vessel, it readily assumes the shape of the interior of that vessel.

Liquids like treacle, honey, pitch, syruphs, etc., offer rather a considerable resistance to forces which tend to alter the shape. Such substances in which the tangential forces between the layers in contact cannot be neglected, are called **viscous** fluids. It is to be noted, however, that a viscous fluid will also yield ***in time*** to any shearing stress.

In the case of a gas we know the fact that if a quantity of a gas be allowed access to any closed vessel, it will fill every nook of that vessel. A gas, therefore, resembles a liquid in ***fluidity*** which results from its inability to resist a change in form.

Fluids.

Hence a ***fluid is a substance which yields to any continued shearing stress, however small.*** In other words, a fluid does not possess elasticity of shape, while a solid does. But a fluid, like a solid, will resist a force tending to reduce its volume and will recover that volume when the force is withdrawn : thus it is said to have volume elasticity (see art 110).

It has been seen that the term ***fluid*** includes both liquids and gases. A gas, however, differs essentially from a liquid in its ***indefinite compressibility*** and its power ***of indefinite expansion.***

The **compressibility** in liquids is so small that for a long time they were regarded as incompressible subs-

tances. The first person to prove with an exact measurement that liquids are compressible was OERSTED, a Swedish Physicist (1822). The instrument that he used is called the *Piezometer*. By means of this apparatus, he proved that all liquids are really compressible under ***very great*** pressures but only to a very slight degree ; thus a pressure, equal to about 200 times that of the atmosphere, will reduce the volume of a quantity of water by $\frac{1}{100}$th. part only. The same pressure will diminish the volume of a mass of mercury by $\frac{1}{1200}$ th. part ; of ether by $\frac{1}{40}$ th. part. As soon as the pressure is removed, the liquid regains its original volume. But in the case of a gas the slightest increase in pressure will at once cause a corresponding change in the volume (Chap. XX).

Again, if a quantity of a gas be enclosed in a vessel and the vessel be enlarged to any extent, it is found that the gas will still occupy the whole vessel : this is certainly not true of a liquid.

A **liquid** is a ***fluid, the volume of a given portion of which never exceeds a definite amount, no matter to how large a space it has access, or how small the pressure to which it is subjected.***

A **gas** is ***a fluid, a given portion of which can be made to expand indefinitely by increasing sufficiently the space to which it has access.***

121. Fluid Pressure.—From the above we may state that the fundamental property of a fluid is that when a fluid is ***in equilibrium***, the force which it exerts on any surface, with which it is in contact, is ***at right angles*** to that surface. The truth of this may be established thus :

Let P, the force, which the fluid exerts on any surface A in it be, if possible, inclined to A and acting in the direction BA (fig. 114) ; when, by Newton's third law of motion, the force which the surface exerts on the fluid is also P acting in the direction AB. The force P may be resolved into a component, R, the normal thrust, acting at right angles to the area A and

a tangential stress T, parallel to the surface. As the fluid cannot resist the latter force, however small it is, the molecules at A will be displaced in the direction of its action, which is contrary to the supposition that the fluid is at rest.

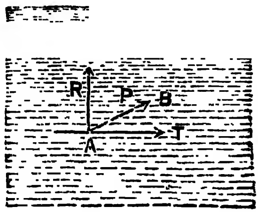


FIG. 114.
Pressure in a fluid.

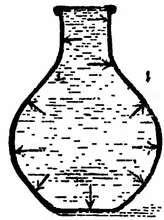


FIG. 115.
Thrust on the sides of a flask.

Hence the force exerted by any fluid at rest on a surface with which it is in contact or on any imaginary place within its mass, which divides the fluid into two, is a normal thrust at right angles to the surface.

Thus in a flask (fig. 115) which is filled with a fluid (water, air etc) the surface will be as shown by the arrow-heads in the figure.

122. Pressure at a Point—The pressure at a point due to a fluid is the thrust exerted by the fluid on a *unit* area surrounding that point.

If the pressure P exerted by a fluid is uniform over a surface of area α, the pressure p at any point is obtained by dividing the total thrust exerted on the surface by the area of that surface, *i.e.*,

$$p = \frac{P}{\alpha}$$

If the pressure be *not* uniform, (as on the vertical side of a vessel containing a fluid mass), so that the force on each equal element of the surface is not the same, the pressure at a point P of the surface is the force which will be exerted on a unit area

at P, if on this unit area the pressure were uniform and the same as it is on an indefinitely small area at P.

The pressure at a point within a fluid is expressed in dynes, or in grammes'-weight per square centimetre, or in pounds' weight per square inch.

123. Pressure within a Liquid.—If a liquid mass be at rest, its weight produces internal pressures in its mass and on the sides of the vessel containing it. Suppose the mass of a liquid is divided into horizontal layers of equal thickness. It is obvious that each layer has to support the weight of the layers above it. Thus the pressure at a point P is *proportional to its depth* (see art. 127).

P varies with its depth.

Expt. 53. Stretch a piece of india-rubber sheet across the mouth of a thistle funnel T and connect it with a glass tube through a length of an india-rubber tubing. A drop W of water introduced within the tube G, will act as an index of pressure (fig. 116). For if the rubber on T is pressed with a finger, the air enclosed in the rubber tube is compressed and the index of water will move forward, the distance moved through being greater as the pressure of the finger is increased. When pressure is taken off, the index reverts to its old position.

Immerse T at different levels below the surface of water. The motion of the index W will indicate that the pressure on the stretched rubber piece increases continually as the depth is increased.

It is also clear that the pressure at the same point depends upon the *density* of the liquid. Thus if a liquid be twice as dense as water, it will exert at P twice as much pressure.

124. Equality of Pressure in all Directions—The pressure at a point within a liquid is exerted in *all* directions about the point and is the *same* in value.

If a plane area be rotated within a liquid mass about a point P, each face of the area is, as we have seen before, acted upon by a thrust perpendicular to the plane. Hence the pressure at a point will have all directions according to the position

of the plane. Thus water rushes into a boat through a leak in the bottom as well as through a hole in the side.

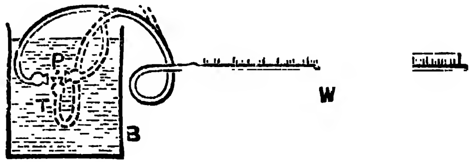


FIG. 116.

Equality of pressures on all sides at P.

Again this pressure is the same in value, whether the surface which receives the pressure faces upwards downwards, horizontally or obliquely.

To show the equality of these pressures, the pressure gauge of Expt. 53 may be used :

Expt. 54. Take the funnel with face downward some depth below the level of water and note the position of the water index. The gauge measures the vertical upward pressure. Now turn the funnel so as to make its mouth vertical, keeping the average depth the same as before. The gauge will now measure the lateral pressure.

Thus turn the funnel mouth in different directions keeping it at the same depth. The almost stationary position of the index shows that the magnitude of the liquid pressure is the same (Fig. 116).

125. The Vertical Upward Pressure —Imagine a horizontal lamina drawn in the liquid mass. The weight of the liquid above it* is the force acting *downwards* on its upper surface. In order that equilibrium may be obtained, there must be an *upward vertical thrust* across the plane, equal to the weight of the liquid above it. This upward pressure is sometimes termed the **Buoyancy** of liquids.

*The pressure of the atmosphere over the surface of the liquid is not considered here.

Expt. 55. Take a long tube closed at the bottom and push it down into water with the closed end downward. On leaving the tube to itself it will be seen to spring upward at once due to an upward pressure exerted by the liquid on the bottom of the tube.

Expt 56. Next immerse the tube in mercury. Greater pressure is now required to push it down. Mercury being denser, the upward pressure exerted by it which opposes the downward motion of the tube is much greater.

Expt. 57. Place a hollow cylindrical vessel, such as a tin can, bottom downward in a vessel of water. The can floats. The buoyancy of the liquid exerted against the bottom of the can supports the weight.

Apply force to lower the can down in water. The upward pressure against the bottom which goes to a lower depth is now increased. As soon as the force is taken off, this pressure which is greater than the weight of the can, pushes it up until the can comes to its former position.

Expt. 58. Take a glass tube, one end of which is ground. Hold a thin plate of flat glass or better a thin card against the bottom of the tube with a string as shown in fig. 117. Lower the whole to some depth into a vessel of water and release the string. The plate does not fall. It is kept in its position by the upward pressure of water.

Pour water (which may be coloured) slowly in the tube. If the weight of the plate be small, it will be found to drop when the height of water inside the tube is almost the same as that of the water outside.

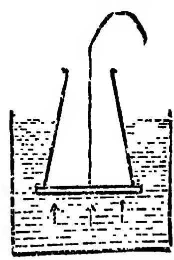


FIG. 117. Upward pressure in a liquid.

This shows also that the upward pressure at the surface is equal to the downward pressure of a column of water standing on the plate (see art. 124).

126. The Lateral Pressure.—The existence of the lateral pressures which a liquid exerts upon the sides of the vessel which contains it, may be understood by supposing that a hole is made on the side of the vessel and that this hole is covered by a plate which exactly fits the hole. The plate will remain at rest only when some external force is applied to it.

When a liquid mass is at rest, the horizontal pressures on the two sides destroy each other, for it is a known fact that a vessel shows no tendency to horizontal motion by being simply filled with a liquid. Accordingly, if we remove an element of one side of the vessel leaving a hole through which the liquid can flow out, the rest of the pressure against this side will be insufficient to

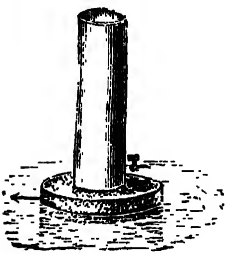


FIG. 118.
Backward movement of a discharging vessel.

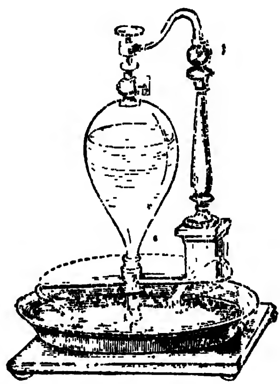


FIG. 119.
Hydraulic tourniquet or Barker's mill.

preserve the equilibrium and there will be an excess of pressure in the opposite direction.

Expt. 59. Fig. 118 represents a floating vessel of water having a stop-cock on one side. Fill it with water and open the stop-cock while the vessel is at rest. Water jets out, and the vessel is seen to move slowly in a direction opposite to that of the issuing stream.

In a *Hydraulic Tourniquet* or Barker's Mill (fig. 119) the effects due to the same cause are more marked. This consists of a vessel of water free to rotate about a vertical axis. This bottom of the vessel is provided with horizontal discharging arms, all bent in the same direction at the ends. Water emerges by the apertures of the bent tubes, and the mill is seen to rotate rapidly.

The unbalanced pressures at the bends of the tubes, opposite to the opening cause the apparatus to roate in a direction opposite to that of the issuing stream of water.

127. The Magnitude of the Pressure—In art. 124 we learn that pressure on a given surface in a liquid mass is, so far as its magnitude is concerned, independent of the direction in which that surface is turned, provided the *average* depth is kept the same. An expression for this pressure may be obtained from the fact that this is equal to the weight of the column of liquid whose base is the given surface, and whose altitude is the average depth, *i.e.*, the depth of the centre of the surface beneath the free surface of the liquid.

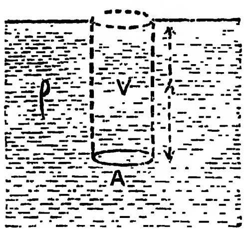


Fig. 120.
Pressure in a liquid.

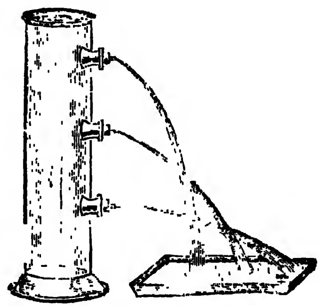


Fig. 121.
Pressure at different depths.

Let A = area of the given surface (fig. 120).

h = mean depth of surface.

ρ = density of the liquid.

g = acceleration due to gravity.

M = mass of volume V of liquid in the column on base A and height h.

Then the total thrust P on A is given by the relation

$$P = Mg = V\rho g = Ah\rho g.$$

The pressure *at any point* on the surface is given by

$$p = g\rho h.$$

If we consider the atmospheric pressure which acts on the surface of the liquid, the pressure p at a point below the liquid level must be increased by the atmospheric pressure, for, as will be seen in art. 130, a pressure applied anywhere on the surface of liquid is transmitted throughout the mass of the liquid. Denoting this pressure per unit area by π, the pressure at a point at deph h is given by

$$p = \pi + g\rho h.$$

From the above relation we find that the pressure at any point in a liquid depends upon

(*a*) the depth of the point

and (*b*) the density of a liquid

Fig. 121 represents a tall vessel with several holes in its sides at different depths. When the vessel is filled with water, water jet is seen to flow out through the holes with different forces, for the pressures inside the liquid at the level of the holes vary, being greater at a greater depth.

128. The Hydrostatic Paradox.—Since the amount of total pressure on a horizontal area A at a depth h in a liquid of density ρ is $Ah\rho g$, it follows that the pressure on the bottom of a vessel containing a liquid is not affected by the breadth or narrowness of the upper part of the vessel. Thus if a number of vessels of different shapes have bases of the same area A, and are filled to the same height h, with the same liquid of destiny ρ, the total pressure on the bases should be exactly the same in all the vessels.

This conclusion is sometimes called the *hydrostatic paradox*; for, at first sight it seems quite impossible that the small quantity of liquid in the vessel C (fig. 122) can exert the same force on the bottom as a much larger amount of liquid in the vessel A. But the following experiment due to Pascal fully verifies the conclusions.

Expt. 60. The vessels A,B,C,D in fig. 122 known as *Pascal's vases* are open at the base and have the same area of the

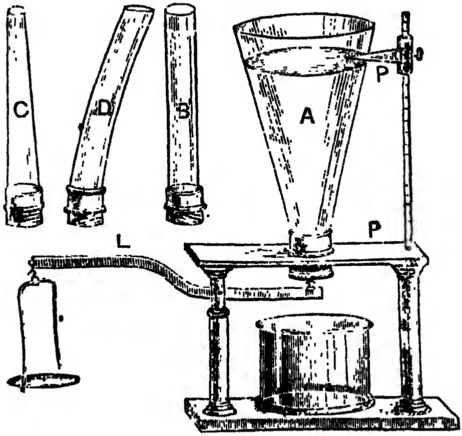


FIG. 122.

Pressure on the base of a liquid vessel.

aperture. They can be screwed on the platform P. A disc attached to one end of a lever L (or a balance) presses against the bottom of any of the vessels when screwed on to the stand. The other arm of the lever or (balance) carries a scale pan on which weights can be placed.

Screw on to P one of the vases. Place a suitable weight on the scale-pan. Pour water within the vase until the disc just begins to give way and allows some water to escape Note the level of water by means of the adjustable pointer T. Remove the vase and replace it by another, and repeat the same experiment. It will be found that water begins to escape, when it attains the same level as before.

Thus the thrust on the base depends on the *head* or the height of the liquid and *not upon* the shape of the vessel or the quantity of liquid contained.

***129. Pressure on the Support.**—It should be noted, however, that in the last article we examined

the thrust on the base of the vessel containing a liquid. But the force on the table on which the vessel stands, does in all cases consist of the weight of the vessel itself, together with the *resultant pressure* of the contained liquid against it. The actual pressure of the liquid against any portion of the vessel is normal to this portion and may be resolved into two components of which we are to consider the vertical components only, as the horizontal components within the liquid destroy each other. It may be proved that the resultant pressure is always equal to the total weight of the contained liquid.

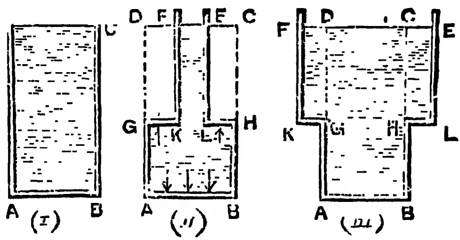


FIG. 123.
Force on the supporting table.

Thus in the case of the cylindrical vessel ABCD (fig. 123), it is obvious that the pressure exerted by the liquid on the bottom is the only force transmitted to the stand and is equal to the weight of the liquid. In the case of the vessel No. (II) the resultant pressure on the stand is the difference of the downward pressure on AB which is equal to the weight of the column of water ABCD, and the upward pressures on the surfaces HL and KG, the latter being equal to the weight of the water which would fill the spaces DFKG, and ECHL. Hence the pressure transmitted to the stand is simply the weight of water in the vessel.

Similarly in the vase No. (III) which is wider at the top, the resultant pressure is the weight of the column of water ABCD which presses on the bottom AB together with the downward forces upon the sides GK and HL equal to the weights of the liquid columns FDGK and ECHL. In other words, it is equal to the weight of the liquid contained in the vessel.

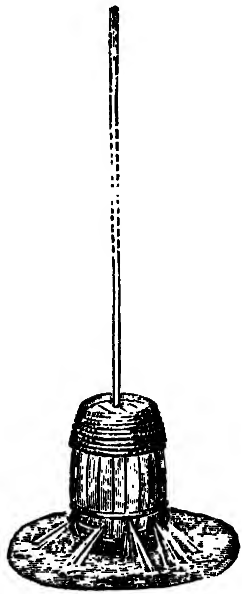


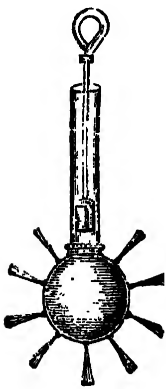
FIG. 124.
Hydrostatic Paradox.

Pascal fixed a long narrow tube about 30 ft. long to the top of a stout cask and then filled the cask and tube with water. It will be understood from the above explanation that though the actual weight of the liquid contained was small, the pressure on the bottom of the cask was equal to the weight of a column of water whose base was the bottom itself and whose height was that of the water in the tube. The result was that the cask burst (fig. 124).

130 Transmission of Fluid Pressure: Pascal's Law.—If the pressure at any point of a fluid is changed, that at all other points is changed also. PASCAL, the great French scientist and mathematician (1623-1662) first made publicly known in his treatise 'Equilibrium of Liquids' that *pressure exerted anywhere on a mass of liquid is transmitted in all directions so as to act with undiminished force per unit area.*

This transmitted pressure does, as we have seen before, always act perpendicularly to any surface in contact with the liquid mass, as otherwise it would have a tangential component, the existence of which is denied by the fundamental property of a fluid at rest (see art 120).

Expt. 61. Fig. 125 represents a glass globe provided with openings on all sides on its surface and a stem in which a water tight piston works. Fill the globe with water. On pressing the piston water will come out in a straight jet from every hole, thus showing that the pressure applied is transmitted through water and acts at right angles to any surface exposed to its action.



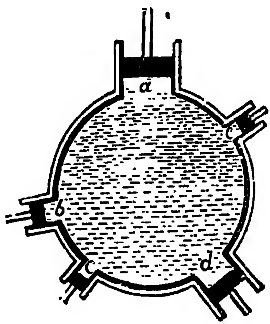


FIG. 125. FIG. 126.

Transmission of Liquid Pressure.

Imagine a vessel having a number of openings of various cross-sections *a*, *b*, *c*, *etc.*, each closed by a water-tight movable piston (fig. 126). Suppose the vessel to be filled with a liquid, and suitable forces be applied to the pistons to keep them in their places. If now an additional pressure p per unit area be applied, so that the total force on the piston of area a is equal to pa, all the other pistons will be forced out. In order to maintain the equilibrium of the liquid mass, it is necesary to apply an additional force to each of the pistons, which

is given by the product of the area of the piston and the increased pressure (p) per unit area. If the pistons are all of the same area, the additional force to maintain each piston in position will be the same.

Expt. 62 Fill a flask completely with water so as to exclude all air bubbles and just to allow a cork fit tightly against its top. Try to force the cork in by hammering it. The flask bursts.

131. Multiplication of Force by the Transmission of Fluid Pressure.—In fig 127, A and B are two cylinders of different sectional areas, say α and β respectively, and communicate with each other through a pipe C. They are filled with water. A downward force w applied to the piston in A will support a large weight W acting on the larger piston in B in order to maintain equilibrium, for the thrust per unit area of the piston in $A = w/\alpha$, which is transmitted to each unit area of the piston in B. Hence the total upward thrust on this latter $= \frac{w}{\alpha}\beta$, which equals W.

or $$\frac{W}{w} = \frac{\alpha}{\beta}.$$

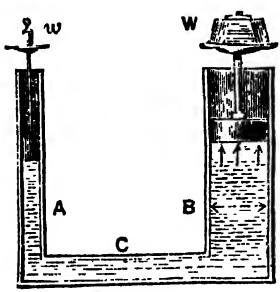


Fig. 127.
Multiplication of Pressure.

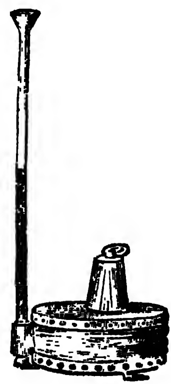


Fig. 128.
Hydrostatic Bellows.

Now if the area of B be 50 times that of A, one kilo applied at A would support 50 kilos at B. Thus a very small force may be transformed into one of a

large magnitude This principle is utilized in the Hydraulic Press (see art. 132).

In a *Hydrostatic Bellows* designed by PASCAL himself the principle of equal transmissibility of pressure in a fluid is illustrated. A stout india-rubber bladder or a leather bellows (fig. 127) is attached to a piece of a tube. The tube is fixed in a vertical position and the bladder rests on the table. A piece of light board is placed on the bladder and a weight rests on the board The bladder and a part of the tube are filled with water. A heavy weight placed on the board will be supported simply by the weight of a column of water in the tube.

132. The Hydraulic Press.—The hydraulic press is in common use wherever great pressure is required. Thus it is applied in compressing paper, cloth, cotton etc. ; testing the strength of iron beams, steel plates for armour-clad vessels ; in extracting oil from seeds ; in making dies, embossing metal etc.

It consists of two iron cylinders or barrels. A and B (fig. 129), the section of B being much greater than that of A. There is in each cylinder a solid rod plunger acting as a piston. The piston in *A* can be move up and down by means of a lever P, having its fulcrum at *K*. In the cylinder *B* there is a large iron ram *R* working as a piston which carries a platform on its top, whereon the objects to be pressed are placed. There is a fixed plate above the platform supported on iron pillars. Two valves, one placed in A, say v_1, another say v_2, in the connecting tube T allow passage to water in one direction only.

As the piston is raised by means of the lever *P* pressure in the barrel A is diminished and the water from the cistern *D* opens the valve v_1 and enters the cylinder. When the piston comes down, the valve v_1 is closed, the second valve v_2 opens and water passes into the large cylinder B. Pressure applied on the piston in A is thus transmitted to the second piston

and is many times multiplied. The platform rises slowly

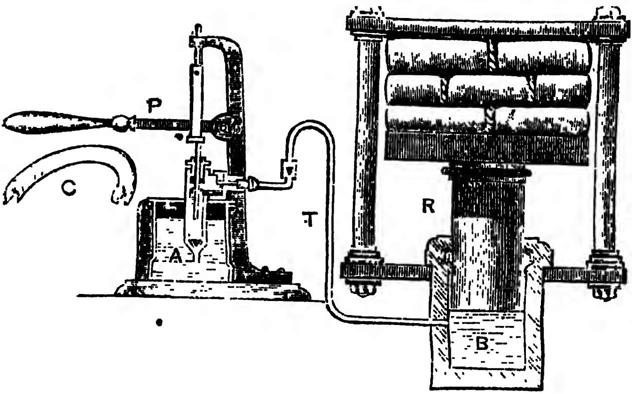


FIG. 129.
The Hydraulic Press.

with a very large pressure. The valve v_2 prevents the water from rushing back from the barrel B.

Let F be the force applied to the lever.

The mechanical advantage $= \dfrac{\text{Power-arm}}{\text{Resistance-arm}} = m$ say.

The downward thrust on the piston in A $= F_1$ say $= m F$, which is distributed over an area α of the piston.

Hence the upward thrust on the larger piston in B

$$= F_2, \text{ say} = F_1 \times \frac{\beta}{\alpha} = m.F.\frac{\beta}{\alpha}.$$

Therefore the mechanical advantage of the whole machine

$$= m\frac{\beta}{\alpha}$$

Thus if the long arm of the lever is 10 times that of the short arm, and the area of the large plunger is 50 times that of the small plunger, the multiplication of force applied $= 10 \times 50 = 500$ times.

Though Pascal illustrated the principle of multiplying a force by hydrostatic action, the apparatus was practically useless for many years on account of the difficulty of making the pistons fully water-tight. BRAMAH got over the difficulty first (1796) by inventing the cupped leather collar which prevents the water from escaping by the sides of the pistons. It consists of a leather ring, semi-circular in section, which is fitted with its concavity downward round the piston in a hollow in the sides of the cylinder (C in fig. 129). Water in passing between the piston and cylinder fills the hollow under the ring and produces by its pressure in it a packing which becomes more tight as the pressure on the piston increases.

It is evident that the decrease in the volume of water in A is equal to the increase of water in B.

Hence $\quad X\alpha = x\beta$

where x is the distance through which the small plunger has to move down, so that the big plunger is raised through a distance x

$$\text{Or} \qquad \frac{X}{x} = \frac{\beta}{\alpha} = \frac{F_2}{F_1}$$

$$\therefore \qquad F_1 X = F_2 x.$$

As *work = force × distance*, the above equation represents that the work done by the large piston is the same as that done on the small piston. Thus in the example given above, the small piston has to come down through 50 times the distance through which the large piston is raised. Again the end of the lever-rod where power is applied, has to be pushed down through 500 times the same distance.

Thus the Principle of work of the Conservation of Energy holds true in this case.

Exercise.—XIV.

1. How would you prove experimentally that a liquid exerts pressure in all directions ?

A tin vessel provided with a tap at the side near the bottom is filled with water and made to stand upright on a thick plate of cork. Explain what will happen when the tap is opened. [*C. U.—1914.*

2. Describe experiments to show that water exerts pressure in all directions.

A plate 10 mm square is placed horizontally a metre below the surface of water, when the height of mercury barometer is 76 mm. What will be the total pressure on the plate ? (Density of mercury—13·6) [*C. U.—1911.*

3. Define intensity of pressure at a point P in a fluid.

Prove that the difference of pressure between the surface of a liquid and a point in the liquid z cm. below the surface, is given by $p=gdz$, where d is the density of the liquid, and g, the acceleration due to gravity. [*C. U.—1910.*

4. Find the pressure due to a column of mercury 50 cms. high.

Does the pressure vary with the diameter of the tube in which the mercury is made to stand ?

5. A laboratory is supplied from a tank at a height of 10 ft. Determine the available water-pressure.

6. A cube, the edge of which is 10 cms. is suspended in water with its sides vertical and its upper surface at a depth of 10 cms. below the surface. Find the pressure on each of its faces.

7. State Pascal's law as to the transmission of pressure in a liquid.

Describe the principle and action of a Hydraulic Press, giving a sectional diagram. [*C. U.—1922.*

8. How will you measure the pressure exerted by a liquid at a specified depth ?

Describe experiments by which you will measure the pressure of the gas and water supplies in the laboratory at the respective taps on your working bench, [*Pat. U.—1919.*

CHAPTER XV.

EQUILIBRIUM OF LIQUIDS

133. Conditions of Equilibrium of Liquids—We have already seen in art. 121 that a liquid mass cannot be at rest in any vessel unless the pressures exerted on a liquid molecule at any point act from all directions and are equal in value. The truth of the above conditions is self-evident, for if the forces exerted on any particle in two contrary directions were not equal, the particle would move in the direction of the greater force, and there would be no equilibrium.

Further, when a liquid is at rest under gravity, as is generally the case, its free surface must be horizontal. If that be not the case, *i.e.*, if some parts, as at A, are higher than others as at C, the pressures at B and D of a horizontal layer BD, taken within the liquid mass are unequal. For, in fig. 130.

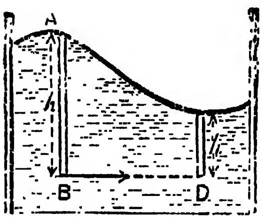


FIG. 130.
The free surface of a liquid at rest.

Press. at B $= \pi + g\rho h$
and Press. at D $= \pi + g\rho h'$
where π is atmospheric pressure, $h = $ AB and $h' = $ CD.

AB being greater than CD, the pressure at B is greater than that at D. Hence the liquid particles which are mobile would move away from places of higher pressure to those of lower ones: in other words, the mass of liquid cannot be in equilibrium.

The state of equilibrium will be attained only when the pressures at B and D are equal *i.e.*, when the liquid surface is horizontal.

If the liquid mass be under the action of gravity and other forces, for example, capillary forces (Chap. XVIII), its surface is inclined, so as to be everywhere perpendicular to the resultant of all the forces which act upon it (see art. 121).

It is to be noted from the above that the pressures at all points in the same horizontal plane are equal. This may also be seen another way :

Take two points A and B in the same horizontal plane (fig. 131) within the liquid. Join AB and consider a very thin cylinder having AB as its axis.

This cylinder is in equilibrium under the action of the following forces :—

(i) its weight vertical and therefore perpendicular to AB

(ii) The thrusts on the curved surface, everywhere perpendicular to the surface and therefore perpendicular to AB, and

(iii) the thrusts on the ends at A and B, these being the only forces in the direction of the axis AB

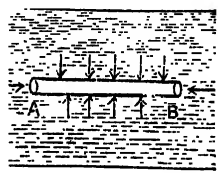


FIG. 131.
Pressure in a horizontal plane.

For equilibrium, the thurst on the end A = that on the end B,

If the ends of the cylinder are very small, we have

pressure at A = pressure at B,

134. Equilibrium of a Liquid in Communicating Vessels—When a liquid is contained in several vessels communicating with each other and is at rest, it stands equally high in all the vessels, so that the free surfaces of all lie in the same horizontal plane. Thus in a tea-pot the water stands at the same height in the spout as in the vessel itself. This fact is commonly expressed by saying that **liquids find their own level.** This is, of course, a consequence of the fact that the pressures in a horizontal layer in the liquid at rest are the same.

Expt. 63.—In fig. 132 the apparatus consists of a series of vessels of different shapes and capacities, connected together by a common tube. Pour water into one of these vessels. The water-level is seen to rise in all the vessels, and will stop at the same height in each.

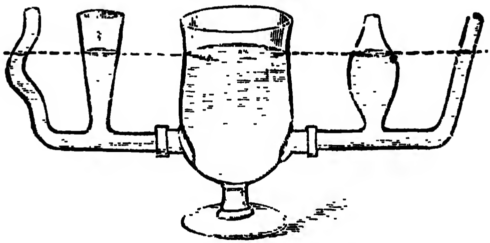


FIG. 132.

A liquid in communicating vessels.

For equilibrium, the pressures at points in the same horizontal plane within the liquid must be the same: again, because the pressure at any point depends upon the height only of the liquid above it and not on the shape of the vessel that contains it (*Cf.* art. 128), the level of water in all the vessels is in the same horizontal surface.

135. Equilibrium of Several Liquids in the Same Vessel—When several liquids of different densities, which do not mix, are placed in the same vessel, they will arrange in the order of their decreasing densities from the bottom upwards.

Expt. 64.—Pour in a long narrow bottle small quantites of mercury, alcohol coloured r d, water saturated with potassiu carbonate (to prevent its m xing with alcohol), and kerosine oil. Shake the bottle and the ı keep it aside on the table.

The liquids are first seeı to mix but gradually they will separate. Mercury, the de sest liquid sinks to the bottom; then come water, alcohol and oil (Fig. 133).

For the same reason fresh water at the mouths of rivers floats for a long time on the denser salt-water of the sea. Cream, which is higher than the milk rises to the surface.

Secondly, the common surface of any two heavy

liquids, which do not mix, is a *horizontal plane*. The statement may be verified by observing the common surface of any two liquids in the bottle (fig. 133), when the liquids are at rest.

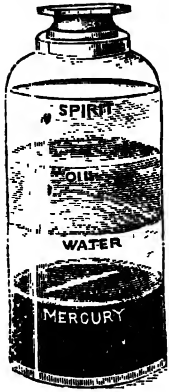


FIG. 133.
Phial of four liquids.

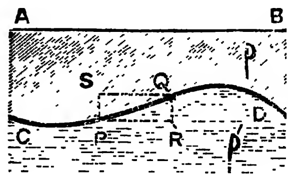


FIG. 134.
Common surface of two liquids which do not mix.

Theoretically the statement may be proved in the following way :

Let P and Q (Fig 134) be any two points on the common surface CD of two liquids of densities ρ and ρ'. Let PR and QS be the horizontal planes through P and Q.

The pressure at P − pressure at S $= g\rho h$

Also pressure at R − pressure at Q $= g\rho' h$

where h = perpendicular distance between the horizontal planes PR and QS.

But pressure at P = pressure at R

and pressure at S = pressure at Q

$$\therefore g\rho h = g\rho' h$$

$$\rho h = \rho' h$$

i.e., ρ, ρ' must be equal or if they are unequal, h must vanish, *i. e.*, P and Q must lie in the same horizontal plane.

136. Equilibrium of Two Liquids in Two Communicating Vessels.—If two liquids of different

densities, which do not mix, be contained in two communicating vessels, their levels are no longer in the same horizontal plane, the level of the lighter liquid being higher.

There will be equilibrium only when the heights of the columns of the two liquids in the two vessels above their common surface of contact are inversely as their densities.

Expt. 65.—Take one U-tube of glass. Pour mercury in this. Note that it attains the same level in both the limbs (fig. 135).

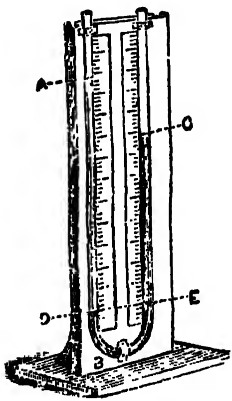


FIG. 135.

U-tube

Pour sufficient water in one leg. Let the two liquids rest with their common surface say at D.

Measure heights AD and CE. Pour more and more water noticing each time the heights of the liquids above the common surface.

Now Pressure at D = Pressure at E, for D and E are in the same horizontal plane.

But press. at $D = \pi + g\rho h$

and press. at $E = \pi + g\rho' h'$

where π = atmos. press., h = AD, and h' = CE ; ρ and ρ' are densities of water and mercury respectively.

$$\therefore \quad \pi + g\rho h = \pi + g\rho' h'$$

$$\text{or} \qquad \rho h = \rho' h'$$

$$\text{or} \qquad h/h' = \rho'/\rho$$

Hence in order that two liquids in communicating vessels may be in equilibrium, their heights above their common surface shall vary *inversely* as their densities.

The relation is made use of in comparing the densities of two liquids (Art. 151).

It is to be noted that as the column of liquid supported in each vessel depends on the pressure per unit area, the cross-section of the tubes does not at all enter into consideration.

137. Level of Liquids.—The surface of a liquid at rest, when taken in small quantity, is regarded to be *level*, *i.e.*, in the same horizontal plane. But the surface ceases to be *plane*, when a large sheet of liquid surface is considered. The horizontal is a direction which is at right angles to the vertical given by the direction of the plummet. As the earth is more or less spherical in shape, the direction of the vertical constantly changes from place to place. A *true horizontal level* is a surface in which all the points are equidistant from the centre of the earth. Hence such a surface is really a curved one. When, however, the liquid surfaces are very small, they may be taken as perfectly level ones in the ordinary sense of the term.

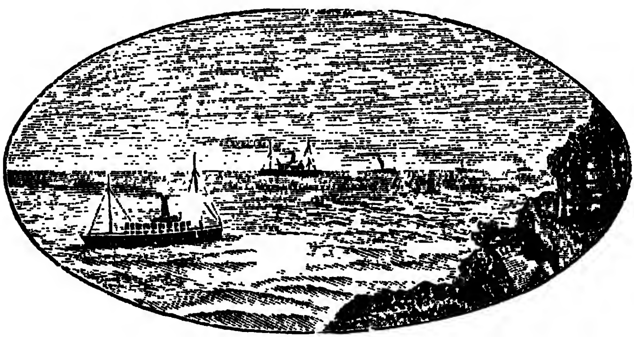


Fig. 136.
The curved horizontal surface of a sea.

The curvature of a liquid surface is easily perceived on a large surface like those of the sea. On observing a ship that is sailing away from the shore, the hull disappears first from the line of sight: then the lower parts of the masts and finally the tops of the masts sink below the horixon (fig. 136). This can happen only when the surface is a curved one.

138. Water-level—This is a simple application of the horizontality of water in two connecting tubes.

It consists of a metal tube (Fig. 137) bent at right angles at both ends, in which are fitted two glass tubes. The tube is placed on a tripod stand which allows the instrument to be turned in any direction. The tube is placed in an approximately horizontal position, and water, generally coloured a little, is poured in the tube until it rises in both of the limbs. When the liquid comes to rest, the surfaces of water in the two legs are in the horizontal plane.

The water-level is used in the operation called the *levelling*, the object of which is to ascertain the difference of levels or the vertical height between two

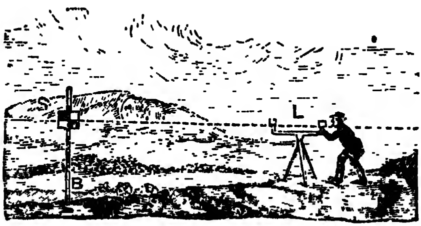


FIG. 137.

Using a water-level.

given points. At each of the points a levelling-staff is fixed. This is an upright graduated rod on which slides a small square tin plate, whose centre serves as a mark for the observer. The observer looks accross the surface of water in the two legs, and directs his assistant to raise or lower the plate until its centre is in his line of sight (fig. 137). The assistant goes next to the other point and repeats the process. The difference of the levels required is given by the difference of the positions of the slide on the staff.

For more accurate work, a telescope with an attached spirit-level is used. The observer reads the graduation on the levelling staff through the telescope which is kept horizontal by means of the level.

139. Spirit-level.—It consists of a glass tube very slightly curved and contains spirit which nearly fills the whole contents of the tube, leaving room for a bubble of air which occupies the highest part. The tube is fixed in a brass mounting (Fig. 138). When the case rests on a horizontal surface, the bubble is exactly between the two lines marked on the glass by the maker. But if the surface be inclined to the horizon, the bubble will always stand nearest the higher end of the tube. By reversing the spirit-level the bubble will change its place in the tube. The instrument, therefore, furnishes a means of testing the horizontality of the line in which the level rests. The spirit-level is more delicate than the water level.

FIG. 138.
Straight Level

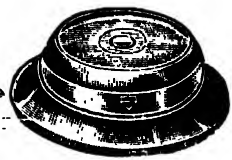


FIG. 139.
Circular Level.

To examine whether a plane surface is horizontal, it is necessary to test by means of a spirit-level that any two directions taken in the plane, *perpendicular* to each other are both horizontal.

Fig. 139 represents the circular form of a spirit-level often used in the laboratories. The spirit is in a small circular vessel with a glass top which forms a part of a sphere, and the under surface of which is concave. A bubble is allowed to remain inside the vessel and a circle is etched on the glass, so that if the circle marked by double is concentric to this, the base of the vessel is horizontal,

140. Water-supply of Towns.—The property of a liquid 'to find its own level' is utilized in the water-supply of towns. A reservoir is constructed on some elevation which is higher than any part

of the district to be supplied. Water is forced up by a pump P into the reservoir, R, which is fed by a river, lake or spring. Main pipes starting from the reservoir, are laid along the principal roads, and smaller service-pipes branch off from these mains to the houses to be supplied. Water will find its way from the reservoir to the houses through the pipes which may rise or fall, in whatever manner is convenient, provided that no part of these is higher than the surface of water in the reservoir.

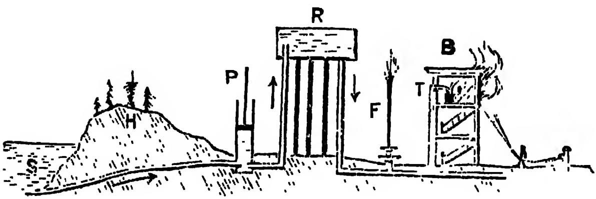


FIG. 140.
Supply of town-water from a river.

In a statical equilibrium *i.e.*, when no water is allowed to escape from the pipes, the tendency of water in the pipes is to rise up to the same level as it has in the reservoir. But, in practice, it does not quite reach this height as. in the fountain F (in (fig. 140) owing to friction of water against the side of the tube and the resistance of air, and to a continual obstruction of energy partly in the shape of kinetic energy of the water that issues from the taps with considerable velocity.

141. Artesian Wells.—The action of Artesian wells (from *Artois* in France) depends on the same tendency of water to find its own level. Fig. 141 represents a section of what the geologists call an artesian basin, The layer A in the earth's crust is composed of some porous materials such as sand, gravel or chalk, through which water can percolate, The strata B and C above and below A are clay, slate or some other material,.

impervious to water. The rain water falling on that part of the part of the stratum A where it comes to the surface, called the *outcrop*, will collect in A. If now a boring is made through the layer B, the water gushes forth to a height that depends on the difference between the levels at the outcrop and of the part where the boring is made.

Many artesian wells exist in the United States, Algeria, Australia, Germany etc. A number of Artesian wells has been bored in the desert of Sahara and an abundant supply of water

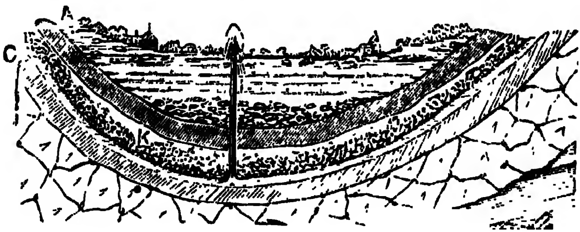


FIG. 141.

An artesian well.

is found at a depth of 200 ft. The waters which feed Artesian wells often come from a distance of a hundred miles. The deepest Artesian well in existence is near Berlin. It is 4194 ft. deep.

Exercise—XV

1. Explain what is meant by saying that *water finds its own level.* Give any practical illustration of this.

2. Describe a spirit-level and how it is used to level a plane surface.

CHAPTER XVI.

PRESSURE ON BODIES IMMERSED IN LIQUIDS

142. Thrust on a Body immersed in a Liquid—It has been shown in art. 125 that a body immersed in a liquid experiences an upward force. A piece of cork, plunged in water and then let go, is pressed up to the surface and floats. It is easy to lift a tub of water within water, but as soon as it leaves the water surface, its full weight is felt. A person bathing in a tank can support his whole weight by pressing lightly against the bottom with his fingers, or may tread upon sharp stones without injury, as he is buoyed up by the water.

Let S (fig. 142) be a body immersed in a liquid at rest under the action of gravity and held in position, if necessary. The solid is subjected to pressures exerted by the surrounding liquid Imagine the body S to be removed and the space ABC, it occupied, be filled up with extra liquid, the rest of the liquid being undisturbed. The pressure on each element of the surface ABC of this additional liquid has the same value as when the solid was in its original position; and

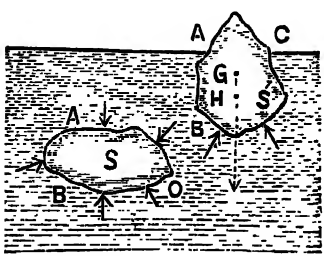


FIG. 142.

Thrust on an immersed body.

hence the resultant thrust on the solid is the same as that on the liquid which has replaced it. Now this latter is in equilibrium under its own weight W acting vertically downwards through its centre of gravity, and the resultant of the pressures due to the surrounding liquid acting on its surface. Hence *the resultant thrust must be equal to the weight of the liquid of the same volume, and must act vertically upwards through the centre of gravity of this portion of the liquid.*

If the solid be not completely immersed, the same conclusion is arrived at by replacing by the liquid that portion of the solid which is below the surface.

The upward resultant thrust on a solid immersed in a liquid, wholly or partially, is called the **Buoyancy** of the body; the centre of gravity of the liquid displaced is the **Centre of Buoyancy.**

The centre of buoyancy of a body does not in general coincide with its centre of gravity. The two points will coincide, only when the body immersed is homogeneous.

The resultant thrust in a liquid does evidently depend on the density of the surrounding liquid and the volume of the immersed body, and is not affected in any way by the nature of the material of the body immersed.

A body experiences no buoyancy, however, unless the liquid acts from below the body. This explains why boats which have settled down on a muddy bank at low-tide do not sometimes rise with the rise of tide.

Expt. 66. Place a piece of wood on the smooth bottom of a vessel, cover it with mercury. Note that it does not rise.

143. Principle of Archimedes.—The deduction arrived at in the last article is known as the Principle of Archimedes, the celebrated Sicilian geometrician of antiquity. The principle enunciates that "*A body immersed in a liquid seems to lose a*

part of its weight, which is equal to the weight of the displaced liquid."

The displaced liquid has evidently the same volume as that of the body immersed; and the loss in weight of the immersed body is caused by the upward thrust equal to the weight of the liquid of the same volume as that of the body.

Archimedes' Principle may be verified experimentally by means of a hydrostatic balance or a spring-balance. A **Hydrostatic Balance** is simply an ordinary balance by which the weight of a body immersed in a liquid is conveniently obtained. In some forms, the balance has one of its pans suspended by shorter suspending frame or chains than the other, and has a hook attached to this pan, from which the body to be weighed is hung by a string. In other forms, as in fig. 143, a wooden stool or bridge C is placed on the floor of the balance-case on one of the scale-pans (say the left-hand one) which can swing freely below it, the supports of the scale-pan passing on either side of the bridge. A beaker can be placed on the bridge, and a body can hang immersed in a liquid in the beaker without interfering with the free movement of the pan, as the beam of the balance oscillates.

Expt. 67.—In fig. 143, A is a solid metal cylinder with a hook attached to its upper end. B is a hollow cylinder closed at the bottom, which just fits on A, so that the interior volume of the cylinder is equal to the volume of A.

Suspend A below B, and B from a hook attached to the knife-edge on the left-hand arm of the beam, so that A hangs inside an empty beaker D on the bridge. Counterpoise the two by placing weights or sand on the other scale pan.

Fill the beaker with water. The upward thrust on A disturbs the balance, and the arm carrying A and B is buoyed up. Drop water from a pipette into the hollow cylinder B. The balance will be restored when B is full.

The upward thrust on A in water is exactly balanced by the weight of water which fills B, and the volume of water in B is just the same as that of A; therefore the buoyancy of A is the weight of water displaced by it.

The experiment may be performed with any other liquid too : the liquid in the beaker D and that which fills up B being, of course, the same.

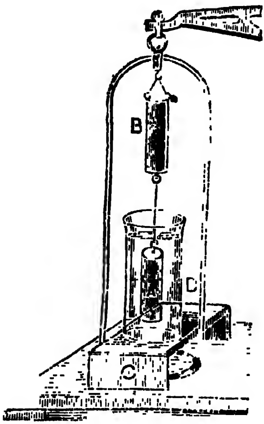


FIG. 143.

Verification of Archimeds' Principle.

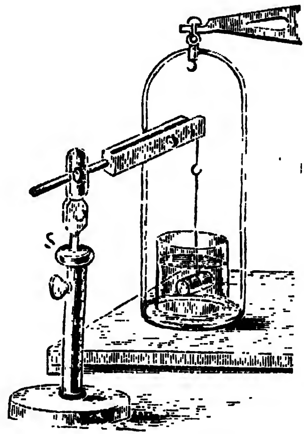


FIG. 144.

Thrust of an immersed body on the bottom of a vessel.

The experiment can be more quickly performed by means of a spring-balance.

The principle can also be verified by taking an ordinary sinker instead of the cylinders A and B.

Expt 68. Get an ordinary glass stopper. Find its weight W in the usual way. Hang it from the suspension hook of the scale-pan by a fine wire and weigh it as it hangs fully immersed in water contained in the beaker D of the last experiment.

Record the apparent weight W′ thus observed, of the body in water. The difference between W and W′ should be very nearly equal to the weight of the water displaced by the body. Now find the volume of the body by the method of displacement of water as explained in art. 15. From the volume in c.c. of the displaced water its weight in grams is known, for the weight of 1 c.c. of water is approximately 1 gm. It will be found that the weight last obtained agrees very closely with the *apparent loss* of weight of the body in water.

It should be noted that the loss in weight of a body immersed in a liquid is only an *apparent* one, for really a vessel of water and a body, say the solid cylinder A in Expt. 67, placed together on the scale-pan of a balance would weigh the same, whether A is outside the vessel or inside it. Though the solid loses a part of its weight while in water, the total weight on the pan remains the same.

Expt. 69. Place a beaker containing water on the scale-pan of a balance and counterpoise it. Thrust your finger in the beaker ; the pan carrying the beaker will go down. Take your finger out (fig. 144).

Take a body K whose volume V c.c. is known by the method of displacement of water. Suspend K in the water from a fixed outside support S. The pan carrying the beaker is again depressed. Place weights on the other pan to restore equilibrium. The weight will be found to be V gms ; but this again is the weight *of the water displaced by the solid.* The immersed body thus seems to add a further weight on the balance-pan equal to the weight of the displaced liquid.

The explanation of the above may be given in the following way :—the solid when immersed in water displaces a quantity of the water of its own volume, and is thereby acted upon by an upward pressure equal to the weight of the water displaced by it. But as actions and reactions are equal and opposite (*Newton's Third Law of Motion*), the body exerts, by way of reaction, a downward force of the same value on the water in the beaker. Hence the supporting pan of the balance is affected by this additional weight which is equal to the weight of the water displaced by the body immersed in it.

The story of Hiero's Crown in connection with the remarkable discovery of Archimedes concerning the weight of bodies immersed in water has often been told. Archimedes (B. C. 287-212) was born at Syracuse in Sicily and lived about the same time as Euclid. 'Hiero, the king had given to an artificer a quantity of gold which was to be fashioned into a crown. When his work was completed, the king found that its weight corresponded with that of the metal which had been delivered to the goldsmith but he suspected that some of the precious metal had been kept back, and its weight made up by baser metals alloyed with the

gold in the crown. He sent the crown to Archimedes to pronounce on the true state of the case. How to do this was for some time a puzzle to the philosopher, but while thinking over the matter, a slight incident suggested the solution of the problem. He was one day entering his bath, which happened to be quite filled, and noticing that the water overflowed its edge in proportion as he immersed his body in the liquid, it struck him that the quantity of water which thus ran out constituted an exact measure of the bulk of the immersed body which displaced it He immediately perceived that if the crown were of pure gold, it would, when immersed in a vessel quite full of water, cause the the same quantity of the liquid to run over the brim as would a lump of gold of the same weight as the crown; whereas the latter, if alloyed with silver or bronze, would displace more water than the lump of gold. When this idea flashed upon the bather's mind, he was so overjoyed at his discovery that he leaped from the bath, and ran home, unclad as he was, crying "Eureka! Eureka! I have found it out! I have found it out." Thus led to make experiments on the weights of bodies in air and in water, Archimedes came to discover an important Principle.

Archimedes on the wt. of a body in water B. C. 250.

144. Determination of the Volume of a Solid by Archimedes' Principle.—Archimedes' principle provides us with the means of finding the volume of body that sinks in water. Let

Weight of the body in air	=	W gm.
Do. do. in water	=	W′ gm.
Then the upward thrust	=	W − W′ gm.

This, by Archimedes' Principle, equals the weight of a mass of water equal in volume to the body.

$\therefore$ the volume of the body = W − W′ c.c.

If the weights are taken in pounds, it must be remembered that 1 cu. ft. of water weighs 62·5 lbs. Hence the volume of 1 lb. of water is 1/62·5 cu. ft. and the volume W − W′ pounds is

$$\frac{W - W'}{62{\cdot}5} \text{ cu. ft.}$$

145. Equilibrium of Immersed and Floating Bodies—Consider a solid of weight W to be completely

immersed in water. Let w be the weight of the liquid displaced. The solid is under the action of two forces, *viz.*,

(1) its own weight W acting downwards through its centre of gravity and

(2) the resultant upward pressure w acting at the centre of buoyancy (see art. 142).

Three cases are now possible :

(1) If $W > w$, the solid sinks in water ; it is denser than water. If it be suspended by a string, the tension T along it must act upwards and is given by

$$T = W - w,$$

The body thus appearing lighter within water than in air. A stone, a lump of iron will sink in water.

(2) If $W = w$, the solid will float wholly submerged and anywhere in the liquid, and will have no tendency either to ascend or to descend.

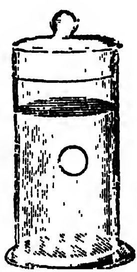


FIG. 145.

Expt 52.—Take a small jar or a bottle. Fill it two thirds with a mixture of equal quantities of water and alcohol. Drop a small quantity of olive oil in the mixture by means of a pipette. The oil collects into a globular shape due to cohesion and should float in the mixture (fig. 145) somewhere within it. If it falls to the bottom, there is too much spirit in the mixture ; if it rises to the top, there is too much water. So this can easily be put right.

(3) If $W < w$, the solid floats partly submerged in water. Cork, wood, wax etc., float on water. As the upward force due to the displaced liquid is greater than the weight of the immersed body, the body is pushed up towards the surface. From the moment it emerges out of the liquid, the weight of the liquid displaced gradually diminishes till it becomes equal to the weight of the solid, when the solid is pushed up no further. It is then in equilibrium unde

Conditions of floating of a body.

the action of the two equal and opposite forces. In practice, the body may execute a few oscillations at the surface of the liquid before it finally comes to be in equilibrium.

Hence *the conditions of equlibrium of a floating body* are :—

(i) A floating body must displace its own weight of the liquid in which it floats. According to Archimedes' Principle a body that floats has lost its whole weight.

(ii) The centre of gravity of the body and that of the liquid displaced are in the same vertical line, for the lines of action of the two opposing forces W and *w* acting on a body in equlibrium must be the same.

Thus a piece of wood weighing 200 gms must displace 200 gms in weight and hence 200 c.c. in volume, of water. It would also displace 200 gms of any other liquid in which it may be allowed to float. If the density of wood be 0·4, its volume is M/ρ or 200/0·4 or 500 c.c. Thus the piece floats in water with $\frac{2}{5}$ ths of its volume immersed.

When water freezes into ice, it expands in volume, and its density is a little less than 1. A piece of ice put in a tumbler of water will float in water and will show only its top above the level of water. In sea-water, the density of which is a little higher than 1 (about 1·03), an ice-berg floats out at water with about $\frac{9}{10}$ ths of its volume under water.

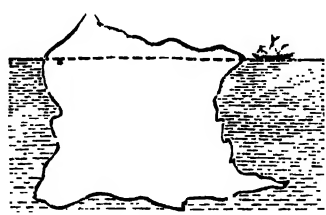


Fig. 146.
An Ice-berge.

A body which floats in one liquid, may sink in another. This happens when the body is lighter than one liquid but denser than the other. Thus floats on water,, but

sinks in ether. A lump of iron floats in mercury but not in water. Similarly, drops of oil float in water, sink in alcohol and swim in a suitable mixture of both. An egg will sink if placed in fresh water and will float, if placed in a strong solution of salt in water. For a similar reason a heavily loaded ship is partly unloaded before it enters a river.

FIG. 147.
The Cartesian Diver.

The different cases of suspension, immersion and floating that can present themselves when a body is immersed, can be shown by means of a well-known hydrostatic toy, the *Cartisian Diver*, described even in old treatises on Physics. The diver consists of small hollow glass ball having an opening in its lower side, by which water can enter or escape. A little porcelain figure is attached to the ball as a counterpoise. Some times the figure has no ball above it, but is itself hollow in the upper part of its body and is provided with a tubular tail open at the end and communicating with the body. The figure is of such a mass that the whole just floats on water with some air in the bulb or in the hollow body.

Expt. 71. Take a jar and nearly fill it with water. Let the diver float in this water. Close the top of the jar with a piece of india-rubber (fig. 147).

Now apply pressure with the hand on the rubber. The diver will sink. On releasing the pressure of the hand, the diver may be made to remain stationary within the water.

What happens is this :—On pressing the india-rubber down, the air above the water in the jar is compressed,

The pressure is transmitted through water to the air in the bulb, and compresses it. More water enters the ball and thereby causes the whole toy to become heavier than the weight of the water displaced by it. Hence the diver descends. On the removal of the hand, the air in the bulb expands, and expels the excess of water which entered it. The figure becomes lighter and ascends. It must be observed, however, that as the diver continues to descend, more and more water enters the ball owing to the increase of the hydrostatic pressure, so that if the depth of the water exceeds a certain limit, the air in the bulb cannot expand sufficiently to allow the diver to rise again, even when the air-pressure on the surface is relieved.

Most fishes have an air-bladder, called the *swimming bladder*, below the spine. By compressing or dilating this at pleasure the fish can either rise or sink in water

A body though denser than a liquid may, however, float on its surface. For this purpose it must be given a hollow or a concave shape, so that if fully immersed, it can displace a volume of liquid, the weight of which is greater than that of its own. Thus a procelain saucer, a boat, a ship float freely on water. Bodies denser than water can also be made to float on water by attaching lighter bodies to them. Hence is the use of life-buoys and life-belts.

Fig. 148 represents a floating dock. When water is allowed to fill the chambers *a*, the dock sinks until the water line is at AA′. The ship is then floated into the dock. When it is in place, the water is pumped out of the chambers until the water-line is as low as BB′. Workmen can then get access to all parts of the bottom of the ship.

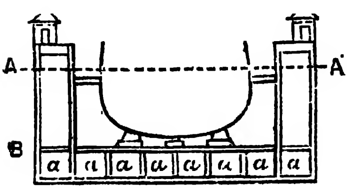


FIG. 148.
Floating Dock.

146. Swimming.—The human body is lighter on the whole than an equal volume of water, the average ratio being 0·934 : 1. Hence it floats on water. In man, however, the head is heavier than water and consequently it tends to sink causing thereby difficulty in breathing. Hence swimming is an art that requires to be learnt, in which practice is to be made in keeping the head above the water surface by muscular action. Air-bladders or cork girdles, known as *Life-belts*, are used by persons who are

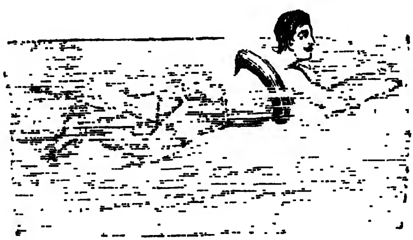


FIG. 149.

Swimming.

learning to swim (fig. 149), as then without any considerable increase in weight, more water is displaced and an increased buoyancy is secured. With quadrupeds, on the other hand, the posterior parts of the body are heavier. Hence they feel no difficulty in keeping their heads above water and thus swim naturally. Several birds such as ducks, swans etc., seem to swim almost on the water surface. This is due to the fact that the thick layers of light down covering the lower part of the body act as a hollow impervious coating, so that even with a very small immersion they are able to displace water of their own weight.

Exercise —XVI

1. State the condition of flotation of a body immersed in a liquid. What volume of water will be displaced by a block of wood of 500 gms weight? (Sp. gr. of wood = 0·65).

2. A lump of metal is known to consist of silver and gold. The lump weighs 20 gms in air and 18·7 gm. in water. How much gold is there in the lump? (Sp. gr. of gold = 19·3; of silver = 10·5).

3. An ice-berg floats with only 500 cubic yards exposed. Find its total volume. (Sp. gr. of ice = 0·918; of sea-water = 1·035).

4. If you were given a piece of wood cut in the form of a cube, how would you very roughly determine its specific gravity without using a balance? [*C. U.—1911.*

5. Explain how Archimedes' Principle may be used to distinguish a metal from its alloy. [*C. U.—1912.*

6. A cubical block of wood of sp. gr. 0·7 floats in water, just completely immersed, when a body of unknown weight is placed on it. Find the weight of this body, if the volume of the block of wood is 100 c.c. [*C. U.—1913.*

7. State Archimedes' Principle. How would you verify it experimentally?

A piece of metal of sp. gr. 8·9 weighs 15·8 gms. in water. Find its volume. [*C. U.—1914*; '20.

8. Why does a solid appear to weigh less in water than in air? [*C. U.—1915.*

9. State Archimdedes' Principle. and explain how it will enable you to identity a given piece of pure metal. [*C U.—1916.*

10 State Archimedes' Principle. How would you demonstrate its truth?

A body weighs 62 grammes in vacuo and 42 grammes in water: finds its volume and specific gravity. [*C. U.—1919.*

11. How would you show experimentally that the resultant vertical thurst on a body immersed in a heavy liquid is equal to the weight of the liquid displaced? [*C, U.—1924.*

CHAPTER XVII.

DETERMINATION OF SPECIFIC GRAVITY.

147. Specific Gravity.—The Specific Gravity (generally shortened to Sp. Gr) of a substance is defined in art 111 to be a *number* which expresses the ratio of the weight of the substance to that of an equal volume of some standard substance.

Thus if W = weight of a substance

W' = weight of an equal volume of a standard substance

σ = sp. gr. of the substance

Then $\sigma = \frac{W}{W'}$.

When determining the specific gravities of *solids* and *liquids*, the standard substance usually taken is distilled water at a temperature of 4°C. Water is suitable for this purpose, for it can be readily obtained everywhere in a pure condition. As, however, the density of water varies with its temperature, a constant temperature such as 4°C is adopted, for it is found that water at this temperature has its greatest density.

The standard substance.

But the variation of density of water due to the change of temperature is very small, so that for all ordinary purposes where great accuracy is not required, it may be assumed that the weight of 1 c.c. of water is 1 gm. that of 1 cu. ft. of water is 62·5 lbs. at any temperature.

Since gases are very light compared to water, the values of specific gravities for all gases would be small fractions, had water been taken as the standard subs-

tance. To avoid this it is usual to adopt hydrogen at a standard temperature and pressure as the standard substance. The density of hydrogen at o°C and 760 mm of pressure is ·0000896 gms. per c.c.

It is to be noted that the specific gravity of a body being a ratio of two weights, is a *pure number*, and does not depend on the units in which the weights are expressed, so long, of course, as the same unit is used for the two. It merely expresses how much a body weighs as compared with water. Thus when it is said that specific gravity of gold is 19, it is meant that volume for volume gold is 19 times as heavy as water.

The Specific Gravity may be defined in other useful forms too. Since the weights are proportional to their masses, we may write

$$\text{Sp. Gr. of a body} = \frac{W}{W'} = \frac{Mg}{M'g} = \frac{M}{M'} \quad \ldots \quad (2)$$

$$= \frac{\text{mass of any vol. of body}}{\text{mass of an equal vol. of the standard substance.}}$$

Again if

$V =$ volume of the substance.

$$\sigma = \frac{M/V}{M'/V} = \frac{\rho}{\rho'}, \quad \ldots \quad (3)$$

$$= \frac{\text{density of the substance}}{\text{density of the standard substance}}$$

Hence sp. gr. is sometimes spoken of as **Relative Density.** In the *C. G. S. system* the density of water is 1 gm. per c.c. Therefore σ equals ρ numerically.

148. Measurement of Sp. Gr.—The specific gravity of a body is generally determined by the follow- methods which require the use of

(1) THE HYDROSTATIC BALANCE.

(2) HYDROMETERS.

(3) THE SPECIFIC GRAVITY BOTTLE. and

(4) THE BALANCING LIQUID COLUMNS.

To measure the specific gravity of a substance we are mainly concerned with the determination of two weights,—the weight of the body and the weight of an equal volume of water. Now the weight of a body as determined by a balance in air is not its true weight, for the body weighed as well as the counterpoise used in weighing will, according to Archimedes' Principle, be buoyed up in the air and will each suffer an apparent loss of weight equal to the weight of the air displaced, and the loss of the weight of the body will not evidently be, in general, equal to that of the counterpoise. But the effect arising from the difference is very small and hence negligible. So in the expression for the specific gravity of a body we shall take the weight of the body in air in place of its true weight in vacuum.

Effect due to the buoyancy in air.

It is to be noted also that in the following articles the expression arrived at for the specific gravity of a body gives the sp. gr. of the substance, relative to water at the temperature at the time of the experiment Had the temperature of the water been $4^\circ C$, the true sp. gr. of the substance at $4^\circ C$ would be given by S, as determined by the experiment. If, however, the water is at any temperature $t^\circ C$, and if the density of the water at $t^\circ C$ is σ times that of the water at $4^\circ C$, it follows that the substance which is S times as dense as water at the $t^\circ C$, is $S\ \sigma$ times as dense as water at $4^\circ C$.

Correction for temperature.

The water which is used for the purpose of the determination of specific gravity should be pure, distilled water free from air.

149. Determination of the Sp. Gr. of a Solid—

(1) DIRECT METHOD.

Expt. 72. Weigh the solid by a balance and let its weight be W gms.

Collect a quantity of water equal to the volume of the solid

by the method of displacement of water (see art. 15). Find the weight of the water thus displaced. Let it be W'.

Then the sp. gr. required $= W/W'$.

(2) WITH THE HYDROSTATIC BALANCE.

The description of the hydrostatic balance has already been given in the art. 143.

(a) *The solid sinks in water and is insoluble in it.*

Expt. 73. Let W be the weight of the body in the usual way. Suspend the body by a fine wire from the hook of one side of the beam; and let it be *totally* immersed in a vessel of water. Let W′ be the apparent weight of the body in water.

Then W − W′

= the upward thrust of the water on the body

= wt. of the water displaced (*by Archimedes' Principle*)

= wt. of an equal volume of water.

Hence $\text{Sp. gr.} = \dfrac{W}{W - W'}$

This gives, as is remarked before, the specific gravity of the solid relative to water at the temperature of weighing and is subject to correction for this

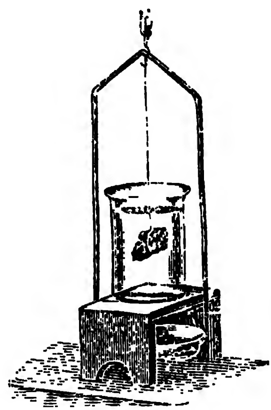


FIG 150.

Hydrostatic Balance.

(b) *Body insoluble in water but is less dense than it*

In such a case, the body must be attached to another body, called a **Sinker**, of a such a kind that the two together would sink in water. This complicates the method slightly.

Expt. 74. Weigh the solid in air. Let it be W. Weigh the sinker alone in water. Let it be w'.

Next weigh the combination of solid and sinker fastened together in water. Let it be W'.

Now suppose w is the weight of sinker in air. This weight need not be found in practice. Then

Wt. of water displaced by sinker $= w - w'$

Wt. of water displaced by combination $= W + w - W'$

$\therefore$ Wt. of water displaced by solid alone

$$= (W + w - W') - (w - w)$$

$$= W - W' + w'$$

$$\therefore \text{ Sp. Gr. } = \frac{\text{wt. of body}}{\text{wt. of equal vol. of water}}$$

$$= \frac{W}{W - W' + w'}$$

We may obtain the expression for the weight of the water displaced by the solid in a simpler way :— When the combination is immersed in water, the upward pressure of water displaced by the solid does not only balance W, the weight of the solid, but is greater than that, so that the excess causes a diminution in w', the weight of the sinker in water, previously determined. In the absence of the sinker the body would have been floated up.

Hence the buoyancy of the solid in water

$$= W + (w' - W')$$

$$= W - W' + w'$$

and the sp. gr. $= \dfrac{W}{W - W' + w'}$.

(c) *Solid soluble in water.*

If the solid be soluble in water, some other liquid must be chosen, in which the solid is insoluble or which is a *concentrated solution* of the solid in water. Let S be the specific gravity of the solid relative to this liquid, determined in the same general way as is mentioned above. If σ denotes the density of the liquid relative to water at 4°C, then $S\sigma$ is the required specific gravity of the solid.

(3) WITH NICHOLSON'S HYDROMETER.

Hydrometer.—A hydrometer is an instrument which is designed to float vertically in any liquid and constructed to determine mainly the sp. gr. of liquids. There are various forms of the hydrometer; but they can all be put in either of two classes. In the type of the hydrometer known as *Variable Immersion Hydrometer*, the specific gravity of a liquid is determined by the depth to which the hydrometer sinks when floating into the liquid; in the type known as the *Constant Immersion Hydrometers*, the hydrometers are always immersed to the same depth in the liquid but carry different weights.

Nicholson's Hydrometer—is a constant immersion hydrometer and is the only one in common use. It is also used to find the sp. gr. of a solid This consists of a hollow body to which is attached a thin stem C, supporting a small pan B above, on which weights can be placed. Below the body hangs a second cup D. This is weighted with mercury or lead, so that the instrument may float vertically in a liquid. On the stem there is a mark to which the instrument is made to sink in the liquid in which it is floating. Thus whatever be the liquid, the volume displaced is always the same.

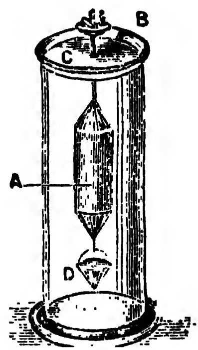


FIG. 151.

Nicholson's Hydrometer.

To determine the sp gr. of a solid substance by Nicholson's Hydrometer, proceed as follows:

Expt. 75.—Float the hydrometer in water in a tall wide jar (fig. 151). Place weights on the upper pan to sink the hydrometer to the index mark. Let W_1 be this weight. Replace the weights in the weight-box and place on the pan the piece of solid whose sp.

gr. is to be determined. The solid must not be big enough to sink the instrument to the index mark. Let W_2 be weight necessary to sink it now.

$\therefore$ The weight of the solid $= W_1 - W_2$

The solid is next placed on the lower pan within the water, where it displaces its own volume of water. As the solid is now acted on by the buoyancy of the liquid displaced, the hydrometer rises a little. Place additional weights on the upper pan until it again sinks to the mark. Let W_3 be the total weight on the pan.

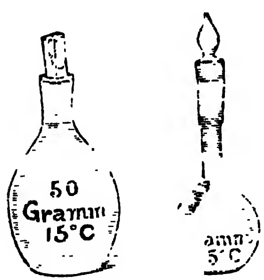
$\therefore$ The weight of water displaced by the solid when in the lower pan $= W_3 - W_2$

$$\text{Hence sp. gr. of the solid} = \frac{W_1 - W_2}{W_3 - W_2}$$

It is to be noted that this method is not so accurate as the other methods are, which involve the operation of weighing by a balance.

(4) WITH THE SP. GR. BOTTLE.

Specific Gravity Bottle—This is a bottle capable of holding a known quantity of liquid. Two forms of bottle are shown in fig. 152. In (i) it is a small flask fitted with a ground glass stopper made from a short length of a thick-walled tubing of a very fine bore. The bottle is filled to the top of the neck with any liquid and the stopper is then pushed home so as to cause the surplus liquid to escape by the hole in the stopper leaving the bottle completely filled. In (ii) the bottle has a narrow neck which is open and has a fine mark on it. When in use, the bottle is always filled exactly up to this mark.



(i) (ii)

FIG. 152.
Specific Gravity Bottles.

The specific gravity bottle is used to find the specific gravity of a liquid ; it is also used for finding the specific gravity of a solid in the form of powder or in small fragments that can be inserted into the bottle, *e. g.*, shots, sand, insoluble powders, metal filings etc.

Expt. 76. Weigh the solid : let the weight be W. Fill the bottle with water and place it with the solid on the same pan of the balance : let the combined weight be W_1. Now place the solid *inside* the bottle and fill it up with water taking care that no air bubble sticks to the side of the solid : let this weight be W_2. Evidently W_2 is less than W_1, as the solid expels water of its own volume.

$$\therefore \quad \text{Weight of water displaced} = W_1 - W_2.$$

$$\therefore \quad \text{Sp. Gr. of solid} = \frac{W}{W_1 - W_2}$$

This result must, if necessary, be corrected for the temperature of the water as explained above.

150. Determination of the Sp. Gr. of a Liquid.

(1) With the Sp. Gr. Bottle.

Expt. 77. The bottle is made thoroughly clean and dried by blowing hot air into it. Weigh it carefully when empty : let this weight be W_1. Fill it with water at a known temperature and again weigh it ; let W_2 be this weight. Finally fill it with the liquid at the same temperature and weigh it : let W_3 be this weight.

Then wt. of liquid filling the bottle $= W_3 - W_1$

wt. of water do do $= W_2 - W_1$

Hence the sp. gr. of the liquid at $t°$ C, relative to water at $t°$ C is given by

$$S = \frac{W_3 - W_1}{W_2 - W_1}.$$

If σ be the density of water at $t°$C, the true sp. gr. of the liquid at $t°$ C with respect to water at 4°C is $S\sigma$.

(2) With the Hydrostatic Balance.

A body which sinks in and is insoluble both in the given liquid and in water, is taken. According to Archimedes' Principle the apparent loss of weight of a sinker

in any liquid gives exactly the weight of the displaced liquid. Hence if the *same* sinker be weighed in the given liquid and in water, the apparent loss of weight in the liquid gives the weight of the liquid of the same volume as of the sinker, and that in water gives the weight of an *exactly equal volume* of water.

Expt. 78.—

Let the weight of sinker in air $= W$

Do do in water $= W_1$

Do do in the liquid $= W_2$

Then the wt. of a quantity of liquid equal in volume to the sinker $= W - W_2$

and the weight of a quantity of water equal in volume to the sinker $= W - W_1$

Hence Sp. gr. $= \dfrac{W - W_2}{W - W_1}$

(3) With the Hydrometers.

Hydrometer of Constant Immersion :—A description of the hydrometer has already been given in the previous article. **Nicholson's Hydrometer** which belongs to the class of the *Constant Immersion* hydrometer may be used to compare the specific gravities of two liquids. When the hydrometer floats in any liquid immersed exactly up to the index-mark on the stem, the weight of the instrument together with the weights on the pan is, by the principle of Archimedes, equal to the upward pressure, *i.e.*, to the weight of the displaced liquid.

Expt. 79.—

Let W be the weight of the hydrometer,

W_1 be the weight necessary to sink it to the index mark in the liquid.

W_2 be the weight required to sink it to the mark in water

Then the weight of liquid displaced $= W + W_1$

and do of water displaced $= W + W_2$

Hence Sp. gr. of the liquid $= \dfrac{W + W_1}{W + W_2}$

Variable Immersion Hydrometer :—A common form of a hydrometer of the variable immersion type is shown in fig. 153. It consists of a hollow glass stem ending below in a glass bulb weighted with mercury, so adjusted as to make the instrument float with the stem vertical. The stem is a cylindrical one and contains a paper scale, the graduations of which are meant to give directly the specific gravities of the liquids in which the instrument is immersed. This type of hydrometers is much in use for commercial purposes.

Fig. 153.
Common Hydrometer.

It is evident that the instrument, when allowed to float in a liquid, will sink the deeper, the less the specific gravity of the liquid, since the weight of the liquid displaced must be equal to the weight of the instrument ; and more and more liquid has to be displaced as the density of the liquid decreases. The calculations by which the markings on the scale are obtained, are based on this fact ; in practice, however, the instruement-makers mark the points by immersing the instrument in liquid of known specific gravities.

If the hydrometer sinks to the mark, say 1 in the stem in water at 4°C, the marks for specific gravities of liquids denser than water will be below the mark 1, and those for lighter liquids by marks above 1. Hence a hydrometer to show the specific gravities of liquids of all densities would have to be inconveniently long. Hydrometers are, therefore, usually made in *sets* to be used for liquids lighter and heavier than water respectively, In the former, the mark 1 (generally marked 1000 for convenience) is pretty near the bottom of the stem and the graduations running up the stem become less towards the top : in the latter, the mark 1, is near the top of the stem and the graduations

increased down the stem. The scale is usually adjusted to indicate a change of 0·001 in the specific gravity of a liquid, and each tenth division is marked. Thus, if a hydrometer float in a liquid immersed to a point marked 850 in the scale of the stem, the specific gravity of the liquid is 0·85. The divisions on the scale are not equal, but decrease in length as the bottom of the stem is approached.

Twaddel's Hydrometer, used in England and *Beaume's Hydrometer*, used on the Continent are both of the class of the Constant Immersion Hydrometer and are graduated with arbitary scales. Hydrometers are often specially graduated for specific purposes, thus an *Alcoholometer* determines the strength of alcoholic liquors, and *Urinometer* of urine, *a lactometer* of milk etc.

(4) By the Liquid Column Methcd.

The specific gravity of a liquid may be determined by balancing a column of the liquid hydrostatically against a column of water, and then comparing the heights of the two balancing columns.

The U-tube may be used when two liquids of which the densities are to be compared, do not mix, *e.g.*, mercury and water. It has already been shown in art. 136 that when two liquids are in equilibrium, their heights above their common surface vary inversely as their densities.

or $$\frac{h}{h'} = \frac{\rho'}{\rho}$$

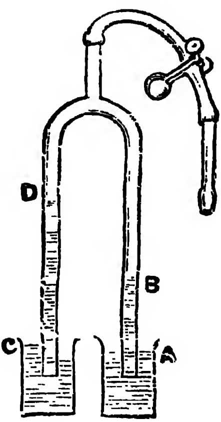


Fig. 154.

Hare's Apparatus.

In the case of two miscible liquids such as water and alcohol, the U-tube may be employed; but a third liquid such as mercury which does not mix with either and at the same time heavier than both,

is to be used. Mercury is first put in the U-tube; water is then poured into one limb and and alcohol into the other, till the level of mercury is the same in both the limbs. The principle mentioned above is then to be applied.

But a more convenient apparatus for comparing the densities of two liquids which will mix, is what is usually known as **Hare's Hydrometer.** It is simply an inverted U-tube dipping into two beakers containing the liquids. At the top there is an opening to which is attached a short length of india-rubber tubing provided with a clip and carrying a glass tube at the other end which is used to draw air out of the tubes. As soon as air is partially sucked out from the tubes, the pressure of air in the tubes decreases. Urged by the external atmospheric pressure on the liquid surfaces in the beakers outside the tubes, the liquids are forced up into the tubes (fig. 154).

Expt. 80. Let ρ and ρ' be the densities of the two liquids in CD and BA respectively; h and h' the heights of the liquid columns, each measured from its own level within the beaker, and P the pressure of air at the top of the U-tube,

then Press. at C = Press. at A

Or $$g\rho h+P = g\rho' h'+P$$

Hence $$\rho h = \rho' h'$$

Or $$\rho'/\rho = \frac{h}{h'}$$

The method of comparing specific gravities by balancing liquid columns is not ordinarily an accurate method.

Exercise.—XVII.

N. B.—In working out examples on specific gravity the formulæ established should never be used. The successive steps in the argument must be worked out from the first principles.

1. Describe carefully any two methods of finding the specific gravity of a piece of glass. [*C. U.—1909.*

2. A prism of cork, 16 cms. high. and of square section equal to 2 cms. side is cemented to a prism of lead of the same cross-section and 1 cm. high. The composite prism is allowed to float in water. How much of it will project above the surface of the water? [Specific gravity of cork, 0·25, specific gravity of lead, 11.] [*C. U.—1910.*

3. A Nicholson's hydrometer weighs 200 gms. and requires 50 gms. in the upper pan to sink it to the fixed mark in water; what weight must be added to or subtracted from the weights in the upper pan to bring it to the fixed mark, when it is placed in a liquid of specific gravity 1·2? [*C. U.—1911.*

4. How would you determine the sp. gr. of a solid? [*C. U.—1913.*

5. A piece of metal weighs 100 gms. in air and 88 gms. in water. What would it weigh in a liquid of sp. gr. 1·5? [*C. U.—1915.*

6. How would you find the specific gravity and the volume of a given solid?

If the sp. gr. of a metal is 19, what will be the weight in water of 20 c. c. of the substance? [*C. U.—1917.*

7. Describe a method of determining the sp. gr. of a liquid.

A Nicholson's hydrometer sinks to a certain mark in a liquid of sp gr. 0·6: but it takes 120 gms. to sink it to the same mark in water. What is the weight of the hydrometer? [*C. U.—1918.*

8. A piece of glass weighs 8·6 gms. in air, 5·85 gms. in water, and 6·4 gms. in alcohol. Find the sp. gr. of alcohol. [*C. U.—1920.*

9. Define specific gravity of a body.

Describe in detail how the specific gravity of a block of alum can be actually determined. [*C. U.—1921.*

10. Distinguish between density and specific gravity of a body.

You are given a piece of paraffin cut in the form of a cube; How would you roughly determine its specific gravity without using a balance?

The specific gravity of ice is 0·918 and that of sea-water is 1·03 ; what is the total volume of an iceberg which floats with 700 cubic yards exposed ? [*C. U.—1923.*

11. The apparent weight of a piece of platinum in water is 60 grammes, and the absolute weight of another piece of platinum twice as big as the former is 126 grammes. Determine the specific gravity of platinum. [*C. U.—1924.*

12 How do you find the specific gravity of a solid lighter than water ?

A piece of cork whose weight is 19 grammes is attached to a bar of silver weighing 63 grammes and the two together just float in water. The specific gravity of silver is 10.5. Find the specific gravity of cork. [*C. U.—1925.*

13. The metal sodium is lighter than water. How would you measure its specific gravity ?

A metal tube, 104 cm. long, 4·1 cm. in external diameter, 3·5 cm. in internal diameter, weighs 100 grams : of what metal would you judge it to consist and why ? [*Pat. U.—1921.*

CHAPTER XVIII.

MOLECULAR MOTIONS AND FORCES IN LIQUIDS.

* **151. Molecular Motions in Liquids**—Experiments with liquids show that their constituent molecules must be considered to be capable of moving continuously from place to place. Evidence of this is found in the very familiar facts of *evaporation* and also in the facts of *diffusion* and *expansion of liquids.*

Further, to account for the phenomena connected with capillarity and surface tension (art. 154), it is necessary to assume that the molecules of a liquid are so close together that the effect of their mutual attraction must be taken into account. In solids, we have seen that this mutual attraction gives rise to *cohesion* ; in gases the average distance between the neighbouring molecules is supposed to be so great, that the effects of this mutual attraction may be left out of account.

Evaporation in an open space.

When a saucer full of water is placed in an open space, water is observed to diminish gradually by evaporation. The molecules of water pass continuousiy into the open space above until the dish is left dry. This phenomenon is difficult to be explained, unless it be assumed that the molecules in the liquid are in motion. During their motion the liquid molecules come into frequent collision with each other, due to which it is reasonable to consider that in a liquid at a constant temperature a molecule may

be moving with greater velocity at one moment than at another. And therefore that at any instant some molecules are moving more rapidly than the rest. Such molecules may, on account of their great velocity break away from the attraction of their neighbours and escape into space above.

Evaporation and temperature.

If it be correct to suppose that the *heat* contained by a body is considered to be the kinetic energy of its constituent molecules, it must follow that an increase in temperature at any time means an increase of the number of molecules which happen to possess at that instant the greatest velocity perpendicular to the liquid surface ; in other words, evaporation ought to take place more rapidly at high temperatures than at low ones. This is, of course, known to be true from our daily observations.

Evaporation in a closed space.

If evaporation takes place in *a closed space*, some of the molecules of the vapour, after wandering about for a time, will strike on the surface of the liquid, and again pass into it. Other molecules will, however, be escaping, and it is clear that after a certain time a state of equilibrium will be reached, in which as many molecules return to the liquid in a second as leave it in that time. The vapour is then said to be in a *saturated* condition.

Expansion.

The same supposition explains the fact of expansion of liquids too. If heat be applied to a liquid contained in a bulb provided with a stem, the level of the liquid in the stem is observed to rise. This is a further evidence of the fact that the velocity of motion of the molecules of a liquid increases with an increase of temperature.

Again in the *diffusion of liquids*, one liquid spreads by molecular motion without the aid of currents in the solutions. If a concentrated solution of copper sulphate be placed in a beaker and water be very

gently poured on it down the sides of the beaker, the two liquids will form two separate layers, the horizontal surface of separation between them being quite sharp and distinct. After sometime, however, it will be found that though the liquids are undisturbed, the blue colour extends upwards indicating that the molecules of the dissolved salt pass upwards; on the other hand, the deep blue colour of the concentrated solution becomes fainter and fainter as it is diluted by the molecules of water passing downwards into the solution.

Diffusion.

Further, the supposition does well explain the *Osmotic* phenomenon in which two liquids which will mix, diffuse into each other even when they are separated by a membrane or a porous diaphragm.

Expt. 81. Fill a sheep's bladder with a strong brine solution. Tie it tightly and then leave it in pure water. It will be found after some time that the bladder is gradually extended to the bursting point; also the liquid outside the bladder has a salty taste.

It thus appears that the molecules of both the liquids pass through the diaphragm in opposite directions but with unequal velocities, the lighter liquid, *viz*, water passing in more rapidly than the salt passing out. Accordingly there is an accumulation of water and hence an increase of pressure inside the bladder.

Osmosis

* **152. Molecular Forces in Liquids**—It has already been mentioned that the mutual attraction of molecules within bodies is powerful only when the molecules are not separated by sensible distances. In liquids, the molecular forces are very small compared to those in solids, but they are not negligible, though that seems to be the case at first sight from the facility with which the liquids change their shape.

Expt. 82. Support a smooth glass plate horizontally from an arm of a balance and counterpoise it. A spring-balance may also be conveniently used for this purpose (fig. 155).

Let the lower surface of the plate come in contact with the clean surface of water contained in a vessel. Now try to detach the plate from the water surface. The force required for this purpose will be found to be rather considerable from the reading of the spring-balance or from the weights necessary to put on the other pan of the ordinary balance.

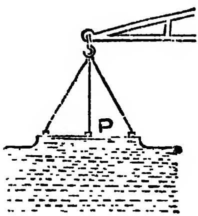


FIG. 155.
Measurement of cohesion of water.

Since a thin layer of water is found touching the glass plate, it is evident that the force applied to pull the plate up has been spent in pulling water molecules away from water molecules, and hence is a measure of cohesion in water. Secondly, the force of adhesion between glass and water is greater than the force cf cohesion in water.

In the case of mercury, the glass will not be found to be wet, showing that the cohesion in mercury is greater than the adhesion of glass and mercury.

* **153. Surface Tension.**—Since every molecule of a liquid is pulling on every other molecule, a molecule such as A (fig. 156), situated well within the mass of a liquid, will be attracted by the neighbouring molecules equally in all directions; whereas any molecule such as B, situated near or on the surface, is attracted by a resultant force directed towards the inside of the liquid mass and perpendicular to the surface. Due to this, the surface molecules of a liquid have a tendency to move towards the interior of the mass, so that the latter may have the smallest possible surface compatible with the volume. In other words, *every liquid behaves as if a thin elastic film, forming its external layer, were in a state of tension,* and exerting a constant effort to contract. The supposition of this tension or contractile force is a convenient fiction which accurately represents the effects of the real cause.

It follows that any mass of liquid would assume a spherical form, as the sphere is the geometrical figure which has the smallest area for a given volume, provided

that we relieve the mass from the action of gravity and other outside forces which, in ordinary cases, mask the presence of the cohesive forces.

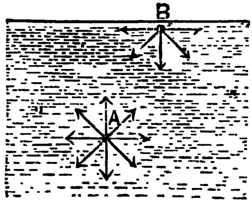


Fig. 156.

Forces acting on a molecule.
A—inside the liquid.
B – near the surface.

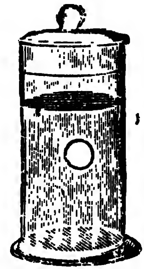


Fig. 157.

Globule of oil freed from the action of gravity.

Expt. 83. Prepare a mixture of alcohol and water such that it has the same density as that of a drop of olive oil. Insert a large globule of the oil beneath the surface by means of a pipette The oil will be seen to float as a perfect sphere within the liquid mass (fig. 157).

By floating the oil in a liquid of the same density as itself (an experiment due to PLATEAU), the force of gravity has no influence on its shape, and the drop owing to cohesion draws itself up into the spherical form.

Again, in small masses of liquids the force of gravity is negligibly small compared with the cohesive forces ; and in such cases the sperical form of the mass is frequently apparent. Thus very small globules of mercury splattering on a table, rain-drops, dew-drops have more or less spherical forms.

That the thin surface layer of a liquid acts as a stretched membrane under uniform tension in all directions may also be understood from the following experiment.

Expt. 84.—Let a sewing-needle be slightly greased and be placed very carefully on the surface of water in a dish. Though it is nearly eight times as dense as water, it will be found to float.

If the needle be previously magnetized, it can be made to move about by means of a magnet held near it.

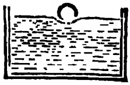


FIG. 158.
A needle floating on water.

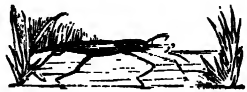


FIG. 159.
An insect walking on water.

Observe the surface in the neighbourhood of the needle ; it shows a slight depression as seen in fig. 158.

The floating of the needle in Expt. 84 seems to be a contradiction of the condition of flotation as deduced from the law of Archimedes (art. 143). But the explanation is quite clear : the depressed portion of the liquid surface has a tendency to straighten out into a flat surface ; the weight of the needle is supported by the vertical components of the surface tension round the edge of the depression. Had the water wetted the needle, as it would have done in the absence of any grease round it, water would have risen about the needle (art. 151) ; the tendency of the liquid surface to flatten out would then have pulled it down.

The above experiment explains the phenomena of insects walking on the surface of water without sinking (fig. 159).

The fact that the surface of a liquid behaves as if it is subject to tension is further illustrated by the behaviour of liquid films.

Expt. 85. Take a bent wire ABC (fig. 160). Allow a thin straight wire simply to rest against this. Get a soap film enclosed in the portion DBE. It will be found necessary to exert a small force to prevent DE from being drawn up due to the contractility of the soap film.

Expt, 86. Dip a flat wire ring in a soap solution and withdraw it. A thin film of the solution will be found stretched across it. Moisten a small loop of thread with the solution and place it gently on the film. It is seen to retain any irregular form that may may be given to it (fig. 161). Now break the film

within the loop ; the loop immediately takes up the circular form (fig. 162).

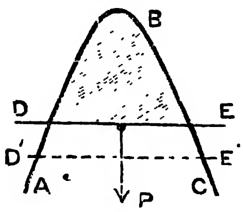


FIG. 160.

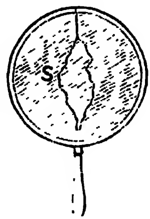


FIG. 161.

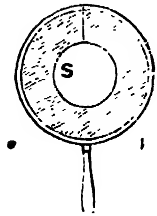


FIG. 162.

To illustrate the contractility of soap-film.

The tendency to contract, of the film outside the loop, is now no longer balanced, as the outer film has vanished. Since the film outside tends to assume the smallest possible surface, the area inside the loop becomes as large as possible, and the circle is the figure which has the largest possible area for a given perimeter.

154. Capillarity.—It was stated in art. 134 that in general, a liquid stands at the same level in any number of communicating vessels. This rule is, however, subject to exceptions in the case of tubes of small diameter (or *capillary* tubes, as they are called from *capillus*, a hair) and in the neighbourhood of the sides of the vessel in which the liquids are contained.

Expt. 87.—Dip glass tubes of different bores in water. As water wets glass, the surface of water round the line of its contact with glass, inside the tube, outside the tube and round the inner surface of the containing vessel is not horizontal but is concave upward (fig. 163). It is to be noted also that water rises higher in the tubes than in the vessel ; and the smaller the tube, the greater the height to which it rises.

Now replace the water by mercury. The effects are found to be just opposite. Mercury does not wet glass and its surface round the line of contact with glass is found to be convex upward. Further, mercury is depressed in all the tubes, the depression being greater in proportion as the bore of the tube is smaller (fig. 164). To see this, bring the tube close to the side of the glass vessel containg mercury. The depression is more easily observed with a U-tube.

Measurement of the diameter of the bore of each tube and of the capillary elevation or depression in it shows that the latter is inversely proportional to the former.

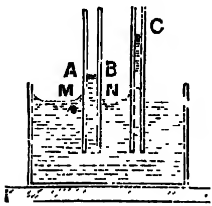


FIG. 163.

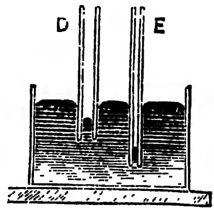


FIG. 164.

Capillary elevation and depression of liquids.

Experiments have established the following laws of capillarity :

1. When a capillary tube is placed in a liquid, the liquid is raised or depressed according as it does or does not moisten the tubes.

2. The elevation in one case, and the depression in the other, are inversely proportional to the diameters of the tube.

3. The height varies with the nature of the liquid, and decreases as the temperature rises.

We now proceed to give an explanation of the effects observed above :—

Suppose the horizontal surface of water meets the glass side at O. Let us consider a small portion of the liquid surface at O. As water wets glass, the force of adhesion between glass and water will pull the liquid particles at O in the direction OG : again the resultant of the cohesive forces within the liquid will pull the same particles in the direction OL. As the former force greatly exceeds the latter, the resultant of OG and OL will be some force OR which lies to the left of the vertical OT. Now since a liquid surface always sets itself at right angles to the resultant force acting,

on it (art. 121), the water surface at O must rise up against the wall, and present a concave surface upward. Further, we have already seen that in small masses of liquids the force of cohesion preponderates over the force of gravity on it (fig. 165).

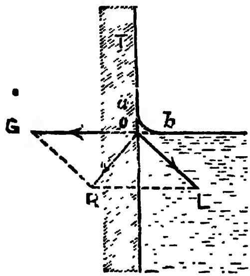


FIG. 165.

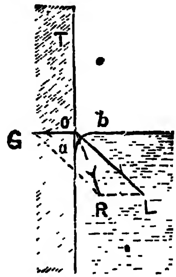


FIG. 166.

Ascension and depression of a liquid surface near a wall.

Conversely, if the cohesive force OL is stronger than the adhesive force OG, as is the case when mercury comes in contact with glass, the resultant OR will fall to the right of the vertical OT, in which case the liquid must be depressed at O and present a convex surface at O (fig. 166).

The above facts combined with the fact that the exposed surface of a liquid always tends to reduce its area explains the ascension and depression of liquids in capillary tubes. In fig. 163 the concave surface *ab* of water inside the tube tends to straighten out into a flat surface due to surface tension. But as soon as it begins to be flat, the forces of adhesion again elevate it at the edges. It is thus evident that water must continue to rise within the tube, until the tendency of the surface to move up is balanced by the weight of the column of water thus raised. We have then

the pull upward due to the surface tension along the line of contact

= weight of the raised liquid column acting downward.

Or $$2\pi r T = \pi r^2 h \rho g$$

Or $$T = \frac{rh\rho g}{2}$$

In the case of water, $\rho = 1$

$$\therefore \quad T = \frac{rhg}{2}$$

Now if the diameter of one tube be half of that of another, the total upward force due to surface tension is reduced to one-half, since the circumference ($2\pi r$) of the liquid surface is reduced by one-half. But the weight of the liquid column of the same height as that in the wider tube, is only one-fourth of the latter, since the volume of the liquid varies as the square of the radius. Hençe, for equilibrium of the liquid column in the narrower tube, the height of the liquid column must be twice that obtained in the wider tube.

In the case of mercury, it must fall owing to the tendency to straighten out of the convex surface of the liquid in the capillary tube The fall continues until this tendency is balanced by the hydrostatic pressure at O.

Instances of capillarity are very common and play rather an important part in our everyday life The rise of oil in wicks of oil lamps, of melted tallow in the wicks of a candle ; the flow of blood through the capillary tubes within the body ; the rise of sap in plants ; the retention of water in a piece of sponge ; the rise of ink in the narrow slit of a pen ; the soaking up of ink by the blotting paper ; the rapid absorption of a liquid by a lump of sugar partially immersed in it are excellent illustrations of capillarity.

Exercise.—XVIII.

1. What is meant by the surface tension of liquids ? Give instances. Enumerate the laws of capillarity.

2. Why is a piece of blotting paper preferred to glazed paper to soak up ink ?

CHAPTER XIX.

PRESSURE IN AIR.

155. Gases—Gaseous bodies possess a number of properties in common with the liquids. Like them, they transmit pressures equally and in all directions according to Pascal's Law ; like them, they possess elasticity of volume only and not elasticity of shape. They differ essentially from the liquids in that they are very much lighter and are very compressible and capable of indefinite expansion (*Cf.* art. 120).

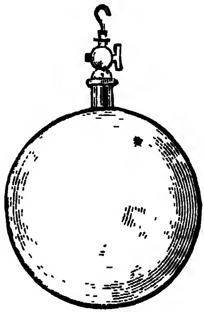
It has, however, been found that the difference between a liquid and a gas is only one of conditions, and depends on the amount of heat in a body. If a liquid be heated sufficiently, it takes the gaseous condition : again, by the aid of pressure and low temperature a gas is convertible into the liquid form.

156. Weight of Air.—In an ordinary observation the air which we take as the type of gases, appears to have no weight. The fact that the air has weight was first proved by OTTO VON GUERICKE, the inventor of the air-pump in 1650, and may be shown as follows :

Expt. 88 A thin glass globe, four or five inches in diameter is provided with a stop-cock and a nozzle by which it can be screwed to the plate of an air-pump (fig. 167). Exhaust the globe as far as possible and determine its weight by means of a delicate balance. Now admit the air by opening the stop-cock and again weigh the globe. The weight in the second case will be found to be greater and the increase in weight is due to the air admitted.

The above fact may also be proved by the following way in which the use of an air-pump is not required.

Expt. 89. Take a fairly large glass flask and close it tight with an india-rubber stopper, through which passes a short tube



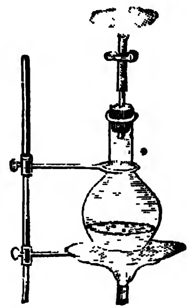


Fig. 167. Fig. 168.

To demonstrate that the air has weight.

carrying a rubber tube and a pinch-cock (fig. 168). Put a little water in the flask, open the pinch-cock and boil the water. As steam comes out, it sweeps out the greater part of the air in the flask. After some time close the pinch-cock quickly and remove the flask to a side to let it cool. Determine the weight of the flask when it is cool.

Now open the clip : the air will be heard to rush in with a hissing sound. Re-weigh the flask carefully. The increase of weight in the second weighing is due to the air that has entered the flask.

Measure the water in the flask by means of a graduated cylinder : fill the flask with water up to the position occupied by the bottom of the stopper, and measure its volume. The difference of these volumes gives the volume of the air which entered the flask.

From the latter part of Expt. 89 the weight of a litre of air at the temperature and pressure at the time of the experiment may be roughly found. Under standard conditions *viz.*, when the temperature is 0°C and the pressure is due to a head of 76 cms. of mercury, a litre of dry air weighs 1·293 gms. Thus in these circumstances the density of air is ·001293 gms. per c.c.

The globe in Expt. 88, after being exhausted of air, may be filled with hydrogen, carbonic acid or any other gas, and their weights may be found in the same way.

157. Pressure of the Air.—Various experiments can be performed to show that air exerts pressure on a a surface in contact with it.

Expt. 90. Depress a beaker, mouth downwards, into water. It will be found that the surface of the water within the beaker is below that outside.

Expt. 91. Close an ordinary small bladder by a string and place it under the receiver of an air-pump (fig. 169). Exhaust the air from the receiver by working the pump. The bladder swells showing that the air inside exerts an outward pressure which was balanced by the pressure of the air in the receiver before the working of the pump. On re-admitting air into the receiver the bladder will be seen to resume its original form.

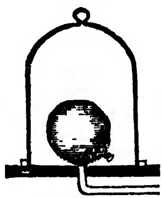


FIG. 169.

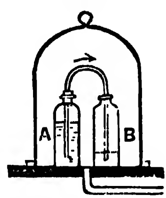


FIG. 170.

Expansibility of gases.

Expt. 92. Fig. 170 represents two bottles placed within the receiver of an air-pump. The bottle A is partially filled with water, stoppered air-tight, and the bottle B is uncorked. A tube passes from A to B, extending almost to the bottom of each bottle. Exhaust the air from inside the receiver ; the enclosed air expands and presses the water in A to pass to the bottle B. Re-admit the air into the receiver ; the water flows back.

The atmosphere encircles the earth as a spherical layer of air which extends, as an appreciable atmosphere, up to a height of some forty miles above the surface of the earth (see fig. 186). The surface of the earth and bodies thereon are subjected to the pressure produced by the weight of the overlaying air. This pressure in the air is called the **Atmospheric**

Pressure. It is subject to the fundamental laws given in the arts. 121-124.

The pressure in the air at any level being due to the weight of the air above, this level must evidently be the greatest at the surface of the earth, and must decrease as we ascend higher in the atmosphere; it has the same value for all points in the same horizontal layer, provided that the air is in a state of equilibrium.

As a gas is easily compressible under pressure, the density of the air too is the greatest at the surface of the earth, and decreases as the height above the surface increases. Whenever there is an inequality of density at a given level due to local conditions, wind must ensue.

Living as we do at the bottom of a deep sea of air, we are not sensible of the pressure existing in the air around us, because it acts with an equal pressure in all directions. In order to make the atmospheric pressure manifest its effects, it must be made to act upon bodies from one side only.

Expt. 93. Stretch a rubber membrane air-tight over one end of an open receiver. Grease the other end, and press it on the plate of an air-pump. As the rubber remains flat, the pressure is evidently the same on both sides of it. Exhaust the air from inside the vessel. The membrane is depressed more and more until it finally bursts under the pressure of the air above (fig. 171); a loud report is caused by the sudden entrance of the air.

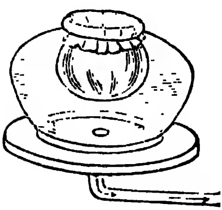


Fig. 171.
Crushing force of the atmosphere.

The experiment may be varied by placing the palm of one hand over the mouth of the open receiver. On working the pump, the weight of the air will be felt at once. As the exhaustion is carried far, it becomes difficult to lift up the hand. Further, the pressure of the air in the tissues of the palm is no more counter-balanced by that of the air

inside the receiver : hence the palm of the hand swells giving a painful sensation.

Expt. 94. Put a small quantity of water into a can of thin sheet tin, and boil the water briskly for some time, so that all the air in the can is expelled by the steam. Now close the can with a good well-fitting cork while the boiling is still going on. Cool the can in a sink by pouring water upon it. The can collapses.

As the can cools, the steam inside it condenses and a partial vaccum is produced inside the can. The walls of the can being not strong enough to withstand the great excess of external pressure are crushed inwards (fig. 172 .

Expt. 95. Fill a glass tumbler quite full with water. Cover its mouth with a sheet of thick paper. Keeping the paper in position by one hand, invert the tumbler with the other. On withdrawing the hand which held the paper, the water will be found not to fall, both water and paper being kept in position by the superior pressure of the atmosphere acting in an upward direction (fig. 173). The object of the paper is to present a flat surface of water, as otherwise the mass of water would divide, thereby allowing the air to enter.

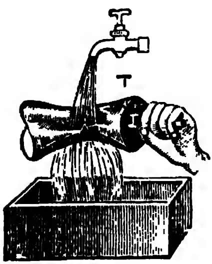


FIG. 172.

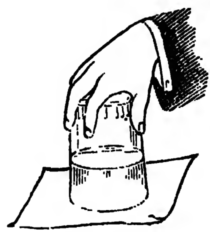


FIG. 173.

Expt. 96. Take a bottle with a very narrow neck. Fill it with water and invert it. Not a drop will be spilt out.

The surface-tension of the drop of water at the mouth of the bottle plays the part of the sheet of paper in Expt 94 ; in other words, it prevents the breaking up of the liquid.

Expt. 97. Fig. 174 represents a tin-can with perforated bottom. Its neck is fitted with a cork through which runs a hole. Fill the can with water. So long as the hole in the cork

is closed with a finger, water will not come out. Water flows out only when communication with the outside air is allowed by opening the hole in the neck.

Fig. 175 represents a toy, called *Magic Bottle*. It is an

FIG. 174.

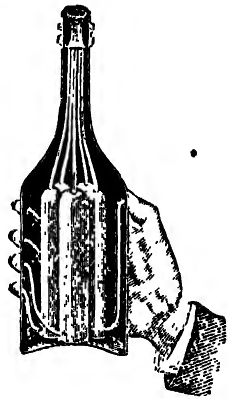


FIG. 17

opaque bottle of sheet iron or guttapercha, containing within it five small vials. Each vial, at its upper part, has a tube which passes up the neck of the bottle ; and at its lower end has another tube terminating in a small hole on the side of the bottle. The five vials are filled with five different liquids. The operator closes the holes by the five fingers of the hand, and pours out at pleasure any liquid out of the bottle by cleverly uncovering the corresponding hole.

The effect of the pressure due to the atmosphere was demonstrated by Otto von Guericke by means of two hollow metal hemispheres. As Guericke was burgomaster of Magdeburg in Prussia, the experiment has always been called the experiment of **Magedeburg Hemispheres**. The hemispheres fit so closely together as to be air-tight, and one of them is provided with a screw to be fitted on to an air-pump. Rings are attached to both the hemispheres at the extreme ends (fig. 176).

Expt. 98. Grease the edges of the two hemispheres and

put them together, note that they are pulled apart easily so long as they contain air. Put these together again; exhaust air from within them, and close the stop cock. Note that a very great force is now needed to separate the two hemispheres.

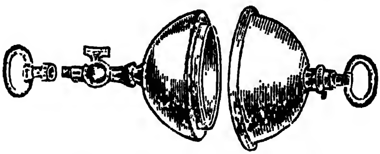


FIG. 176.—Magdeburg Hemispheres.

In one of Guericke's original experiments in 1654, it is said that hemispheres of 2 ft. diameter were used, and that a team of 12 horses, six on each side, was required to pull the hermispheres apart!

158 Rise of Liquids in Exhausted Tubes.—Before the time of Galileo the rise of water in a tube when air is exhausted from its upper end, as in pumps and siphons, was explained by the supposition that *Nature abhors vacuum.* In 1640 Galileo's attention was drawn to the fact that some pumps erected in the garden of the Duke of Tuscany near Florence, designed to draw water from a depth of 56 ft. did not work; the water rose about 30 ft. but would rise no higher demonstrating as it were, that Nature's abhorrence had its limits. Galileo seems to have suspected that the pressure of the air was responsible for the phenomena, but he died in 1642 without being able to prove it. The true explanation was, however, given by his friend and pupil, Torricelli (1608-1647), who took up the question and continued the investigation after Galilao's death.

159. Torricelli's Experiment..—Torricelli argued that if water rose to a height of about 30 feet, mercury which is $13\frac{1}{2}$ times denser than water. must rise to a height of about 27 inches. In 1643, he came to devise an experiment which has immortalized the name of its author.

Expt. 99. Take a thick-walled long glass tube, about a metre long and a centimetre in diameter and closed at end.

Fill it with clean, dry mercury, care being taken to expel all traces of air from the tube (fig. 177). For this purpose, close the open end of the tube with the thumb leaving a small quantity of air above the mercury : then incline the tube gently so as to allow the air bubbles to pass from end to end, thus including in it the small bubbles of air that may adhere to the glass.

Now fill the tube *completely*, and close the open end with the thumb so as not to allow any air bubble between it and the mercury. Holding the tube firmly, invert it and immerse the open end in a small cistern of mercury (fig 177, ii). On removing the thumb, mercury will be seen to descend in the tube, and after a few oscillations come to be stationary at a height which is generally about 76 cms , or nearly 30 inches * above the surface of mercury in the cistern.

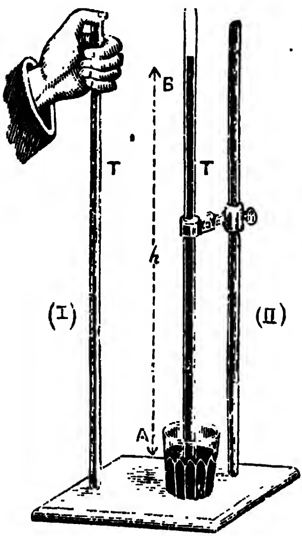


FIG. 177.
Torricelli's Experiments.

We now proceed to find the explanation of the support of the mercury column in the tube, T. The clear space above B, the surface of mercury in T, is devoid of air, and hence a vacuum,† usually called the **Torricellian Vacuum.** Hence the pressure at any point A, *inside the tube*, at the level of the surface of mercury in the cistern is that due to the column AB of mercury. This must equal the pressure at a point on the surface of mercury in the cistern *i.e.*, at the same level as A ; and this latter

* 76 cm = 29·992 in.

† It is not a complete vaccum, for it contains mercury vapour the pressure of which, however, at ordinary temperatures is practically inappreciable.

pressure is due to the atmosphere. It follows that the atmospheric pressure at the surface of mercury outside the tube is equal to the pressure due to the column of mercury, AB, standing in the tube ; in other words, the downward pressure of the atmosphere on the mercury surface in the cistern maintains the vertical column of mercury in the tube at the height observed.

The simple apparatus of Torricelle's experiment constitutes a simple form of a **Barometer** or an instrument for measuring the atmospheric pressure.

The above explanation gets further arguments in favour of it from the following experiments :—

Expt. 100. Fit up a barometer and fix the tube vertically by means of a suitable stand. Note the height of the mercury in the tube, above that in the reservoir. Then incline the tube at various angles to the vertical (fig 178) and measure the vertical height in each case. It will be found that the *vertical height* of the top of the column above the mercury in the reservoir

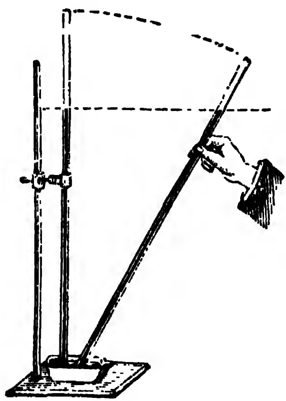


Fig. 178.

Barometric height.

Fig. 179.

Fall of mercury with reduction of air-pressure.

is always the same. It is instructive to note also that if the tube be so much inclined that mercury may strike against the top, there would be a sharp, metallic sound.

Expt. 101. Arrange a barometer so that the trough is under the receiver of an air-pump (fig. 179), the tube passing through a tightly fitting rubber stopper in the bell-jar Exhaust the air gradually. As the pressure on the mercury in the trough is being reduced, the level of the mercury column falls. On re-admitting air into the bell-jar, the mercury rises to the original level in the tube.

160. Pasical's Experiment.—Torricelli observed that the cause of the pressure of the air must be the same as the cause of pressure of liquids viz., the weight of fluid itself, but he died in 1647. before he had the opportunity of submitting the problem to a test. This test Pascal undertook. He reasoned that since the pressure in the liquid diminishes on going up towards the surface, atmospheric pressure also ought to diminish on passing from the sea-level to a mountain-top. He requested his brother-in-law Perrior who lived near Puy de Dome, a mountain in the south of France, to try Torricelli's experiment. In 1648, two observations were made, one at the foot of the Puy de Dome, and the other at the top, a height of about 3565 ft : the mercury column stood at 28 inches at the bottom and 24.7 inches at the top.

161. Amount of Atmospheric Pressure.—It has been proved in the previous articles that it is the atmospheric pressure that supports, and hence is measured by the column of mercury standing in the barometer tube. To calculate the atmosperic pressure therefore, find the hydrostatic pressure at the bottom of a cylinder of mercury of one sq. cm. in cross-section and 76 cm. in height, since that is the average height of the column at sea-level and taken as the *standard* one.

Volume of mercury standing on unit area = 76 c. c.
Now weight of 1 c.c of water = 1 gm
and density of of mercury = 13·596
∴ Weight of 76 c c. of mercury = 76 × 13·596 gm.
= 1035·6 gm.

Thus the pressure is equal to the *weight of 1033 gms. per sq. cm.* or roughly 1 kg. per unit area, and hence this is the intensity of the atmospheric pressure.

To express the normal atmospheric pressure π in dynes per sq. cm., we have $\pi = h\rho g$ where

h = barometric height = 76 cm.
ρ = density of mercury = 13·596
and g = acceleration due to gravity.

$\therefore \quad \pi = 76 \times 13{\cdot}596 \times 981 = 1{,}013{,}663$ dynes.

Again, to express the atmospheric pressure in pounds per sq. inch, we consider a barometric column of 30 inches height and 1 sq. inch in cross-section; this column occupies a volume of 30 cub. inches.

Now 1 cu. ft of water weighs		62·5 lbs.
$\therefore$ 1 cu. in. „ „		$\frac{62{\cdot}5}{1728}$ lbs.
and 1 cu. in. of mercury „		$\frac{62{\cdot}5}{1728} \times 13{\cdot}596$ lbs.
		= ·49 lbs.
$\therefore$ 30 cu. in „ „		$30 \times {\cdot}49 = 14{\cdot}75$ lbs.

Hence the atmospheric pressure is equal to *the weight* of 14·75 lbs., or roughly 15 lbs. per sq. inch

If a gas or a liquid acts in such a manner as to exert a pressure of 15 lbs. per square inch of a surface exposed to it, the pressure is often spoken of as that of **one atmosphere.**

Since in the above calculation the pressure per unit area only has to be considered, the height of the column AB in the baromatric tube is quite independent of the form and area of the cross section of the tube, as well as the extent of surface of the mercury in the cistern.

If follows from the calculations given above that a surface of a foot square is subjected to a pressure of 15×144 or 2,160 lbs. or nearly a ton Now the surface area of the body of a man of middle size is about 16 sq. ft.; hence the pressure on his body amounts to an enormous pressure of 37,500 lbs. or upwards of 16 tons. We are, however, not sensible of this pressure, because at every point it is exerted equally in all directions. At the same time, it is evident that the body, being

Air-pressure on the body.

subject to a normal pressure at all points in its surface, is compressed to an extent which depends upon the elasticity of volume of its component parts ; the solid parts of the skeleton can resist a far greater pressure ; as to the liquids in the organs and vessels, they are virtually incompressible ; the internal air, too. being compressed by the weight of the atmosphere, is under the same pressure as the outside air. The compressing effect of the air pressure on the tissues of the body is one of the conditions to which the structure of the body is specially adapted, and we are not sensible of the effects, because we are always subject to them.

In balloon ascents, and on very high mountains. travellers experience a strong pressure of blood towards the nose and eyes, owing to the fact that the pressure of the enclosed air preponderates over the greatly diminished pressure of the surrounding outer air.

162. Barometers.—By fixing the Torricellian tube (art. 159) in a permanent position, we obtain a means of measuring the amount of atmospheric pressure at any moment : and this pressure is measured by the weight of the column of mercury which it supports. Such an instrument is called a **Barometer** (*Gr. Baros*, weight).

In constructing a mercurial barometer the mercury used must be pure and clean. Further, to drive out air and moisture, the mercury must be carefully boiled in the glass tube. Ordinary mercurial barometers are either *Cistern* or *Siphon* barometers. The barometer may be filled with any liquid other than mercury, for instance water, glycerine : but in that case, the tube would be inconveniently long, as the liquid used is less dense than mercury. In the *Aneroid* barometer no liquid is used ; it is not so accurate as a mercurial barometer, but is light and portable.

Cistern Barometer.—The Torricellian tube standing on its basin of mercury is a cistern barometer. The atmospheric pressure is given by the height of the mercurial column measured always from the surface of

mercury in the cistern to the top of the mercury in the tube (fig. 177, ii). With the variation in intensity of the atmospheric pressure, mercury is sometimes forced from the cistern into the tube, and sometimes from the tube into the cistern. Hence if a scale be permanently fixed by the side of the tube, the zero mark of which is meant to be at the level of the mercury in the cistern which, however, is not constant, an error creeps into the result of measurement, called the *Capacity Error.* By making the area of the tube small in comparison with that of the cistern, the rise and fall of mercury level in the latter can be made small; still for accurate work, it is necessary to allow for this in the graduations.

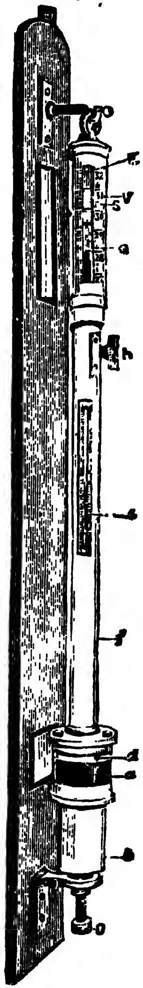


Fig. 180.

Such a barometer is not convenient for use; the cistern must be connected with the tube, and the whole must be protected.

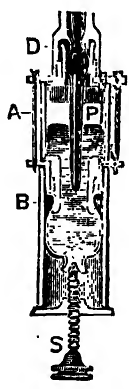


Fig. 181.
The cistern of a Fortin's barometer.

Fortin's Barometer—The most convenient form of mercury barometer for general use in accurate work is Fortin's, shown in fig. 180. It is an improvement upon an ordinary cistern barometer in as much as an arrangement is herein made, so that the zero mark of the scale may radily be brought to coincidence with the surface of the mercury in the cistern.

The usual arrangement of the cistern and the lower part of the tube of the instrument is shown in fig. 181. The cistern consists of a glass cylinder, A, which allows the mercury to be seen ; the bottom of the cylinder is cemented to a boxwood cylinder, B, to which is fixed a leather forming the base of the cistern. The leather is provided with a smalll wooden bottom C, against which the screw S works, thus lowering or raising it as desired. The cistern is enclosed in an outer metal case in the way shown in figs. 173 and 174, allowing the surface of the mercury to be seen through the glass cylinder. The screw S works through the bottom of this outer case. Fixed to the lid of the cistern is a small ivory pin P ; the pointed lower tip of this pin marks the zero of the scale on which the height of the barometric column is read.

The barometer tube drawn at the open end fits through a boxwood collar D in the cover ; the cistern is closed in the upper part by a buck-skin tied to the tube and the outer case ; this strip of leather prevents the escape of mercury from the cistern but transmits atmospheric pressure by allowing free access to the air through its pores.

The tube of the instrument is encased in a long *brass tube f*. At the top of this case, there are two longitudinal slits about 20 cms. long, cut parallel to the length of the tube and diametrically opposite to each other, so that the level of the mercury may be seen. The scale to read the height is engraved on the outer tube along the edge of the front slit; the scale S is ususally graduated from about 27 to 32 inches only, as under ordinary circumstances the variations in the atmospheric pressure do not go beyond this range. A sliding vernier V which can be moved up and down in the rectangular slit cut in the case by a rack and pinion movement worked by a screw *k*, reads up to .002 of an inch. At the lower part of the case is affixed a thermometer *t*, to indicate the temperature.

To read the barometer, the screw is moved up or down until the level of the mercury in the cistern just

comes to touch the tip of the ivory pin. The vernier is next adjusted, until the top of the mercury column, the lower edge of the vernier in front and that of the plate of brass at the back all appear in the same line. The reading is then taken off the vernier and is further subjected to certain *corrections* or *reductions* which are important for an accurate determination of the atmospheric pressure.

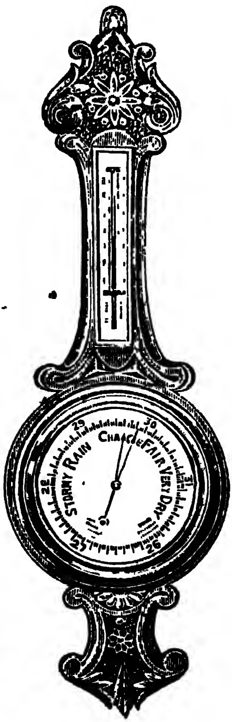
Siphon Barometer.—This is more convenient and portable than the cistern barometer. It has no cistern, but consists of a long glass tube, the open end of which is bent upwards (fig. 182), so that the short open limb BD takes the place of the cistern. The long leg AB which is closed at the top, is filled with mercury, as in the cistern barometer. The difference of the levels of the mercury at A and C, in the closed and open limbs respectively of the tube, measures the height of the barometer.

To protect the mercury surface at C, the end of the shorter arm may be closed leaving a pin-hole D at the side through which the communication is kept with the atmosphere.

Weather-glass or Wheel Barometer.—The ordinary weather-glass or house-hold barometer is a form of siphon barometer. In the shorter leg, there is a float which rises or falls with the mercury. The float is connected by a rack-and-pinion arrangement to a central wheel, to the axis of which is fixed a needle moving on a dial (fig. 184,) The dial is mounted in the front of the tube so as to conceal its presence. It is graduated and marked *stormy*, *rain*, *variable*, *fair* etc. (fig 183). When the pressure varies, the float rises or sinks, and moves the index needle to the corresponding points on the scale. The rack-and-pinion arrangement is sometimes replaced by a pulley and a string carrying a counterpoise at the other end. The wheel barometer is a very old invention and was introduced by Hooke in 1683.

The weather-glass is neither very delicate not very precise in its indications. Further the indications on

the dial as to the state of weather are strictly of any value for that place only for which the barometer is



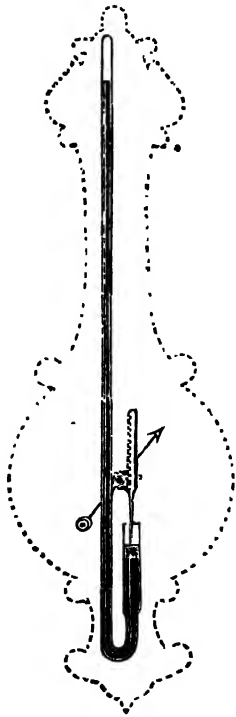


FIG. 182. Syphon Barometer.

FIG. 183. Weather-glass.

FIG. 184. Wheel Barometer.

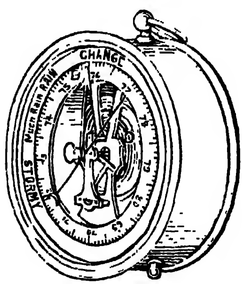
made, and differ for places at different levels and of different climatic conditions.

Glycerine Barometer.—The vapour of pure glycerine has very low pressure at ordinary temperatures and

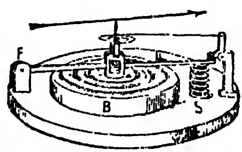
hence there is not much objection to the use of glycerine for barometric purposes as there would be to that of water. The column of glycerine (sp. gr. 1·28) corresponding to 30 inches of mercury (sp. gr. 13·6) is 318·5 inches or about 27 ft. so that the fluctuations of the barometer are magnified about 10·7 times. As, however, glycerine readily attracts moisture from the air, it is usual to cover the liquid in the cistern with a layer of paraffin oil.

The Aneroid Barometer.—This form of barometer is commonly used by geological and surveying parties, as it contains no liquid (from Gr. *a*, not, and Gr. *neros*, moist), and is hence more convenient to carry.

It consists essentially of a small chamber in the form of a cylindrical box (fig. 185). The box is partially exhausted and closed with a diaphragm of thin, elastic metal which is corrugated in order to make it yield more easily to external pressure. Variations



(*a*)



(*b*)

FIG. 185.—Aneroid Barometer.

in the atmospheric pressure cause the diaphragm to yield to an amount proportional to the change of pressure. In some forms the chamber takes the shape of a thin-walled metallic tube in the form of a crescent which is closed and exhausted; the ends of the tube separate or approach as the external pressure diminishes or increases. The motion of the diaphragm or the ends of the tube, as the case may be, is multiplied by a delicate system of levers and transmitted to an index moving over a dial.

whose readings are made to correspond to the readings of a mercury barometer.

The aneroid barometer is lighter, portable and less liable to injury than a mercurial barometer : it is sometimes made small enough to the size of a watch. Changes in temperature may produce some alterations in the readings which should for this reason be checked by the occassional comparison with a standard mercurial barometer.

An aneroid barometer can be arranged to record its indications on a piece of squared paper by means of a pencil fitted to a long lever ; the paper is wound on a cylinder, rotated by a clock work, the whole arrangement is then called a **Barograph** or a self-registering barometer.

163. Variation in the Atmospheric Pressure — When the barometer at a place is observed for several days, its height is found to vary during the same day as well as from day to day. The extent of these variations is from 78 to 71 cms., or 31 to 28 inches.

The above shows that the atmospheric pressure is subject to variations. It is further observed that when the temperature rises, the barometer falls, and *vice versa.* The *daily variations* appear to result from the change of density of air, consequent on the expansions and contractions produced in the atmosphere by the heat of the sun. Whenever there is a difference of temperature in any portion of the atmosphere and its neighbouring parts, currents in the air are produced ; the air from the warmer region rising up and passing away through the upper regions of the atmosphere : thus the pressure in the portion is diminished and the barometer falls.

In the Equator, and between the tropics, the daily variations are rather regular ; the barometer sinks from midday till towards 4 P. M. ; it then rises and reaches its maximum at about 10 P.M It then sinks again and reaches a second minimum towards 4 A.M. and a second maximum at about 10 A.M. Barometer changes are much greater and more rapid in the temperate zones than in the tropical regions.

The use of the height of the mercury column in a barometer,—rather of the changes in the height,—as to the prediction of the possible state of the weather is wide, as a change in weather has been frequently found to coincide with a change in the pressure. But the indications will differ according to the meteorological conditions of a place. As a general rule, however, the following is interesting to note.

Weather indication. As dry air is heavier than damp air, (the density of water vapour being 0·6 of the air at the same temperature and pressure), the barometer rises ordinarily in dry air, indicating fair weather, and falls in moist air which generally precedes a rainy weather.

From the coincidence observed between the barometric heights and the state of the weather the following indications may be roughly remembered,—

HEIGHT	STATE OF THE WEATHER
79 cm ; 31 in	Very dry
78 cm ; $30\frac{2}{3}$ in	Steady
77 cm ; $30\frac{1}{3}$ in	Fair
76 cm ; 30 in	Variable
75 cm ; $29\frac{2}{3}$ in	Rain or wind
74 cm ; $29\frac{1}{3}$ in	Much rain
73 cm ; 29 in	Storm

Further, a rapid rise of the barometer on any day signifies a fine weather, but not lasting : a slow movement or a steady height states the contrary. A sudden and rapid fall, on the other hand, indicates storm ; a slow continuous fall continuing for days together implies a lasting bad weather

The Indian Government has Observatories, in all the principal cities where, along with other observations, the barometric height is noted every day at 8 A.M , 10 A.M , and 4 P.M. The 8 A.M. observations are telegraphed at once to Simla, Calcutta, Bombay and Madras. At these central observatories maps are daily prepared and published. In these maps are drawn the lines, called **Isobars**, connecting places of equal barometric pressure

and curves, called *Isothermal lines*, passing through places of equal temperature. The strength and direction of the wind, and the state of weather and of sea are also entered herein. from these, a meteorological forecast of weather is issued daily.

164. Measurement of Heights by the Barometer.—The barometer may also be used for the measurement of height above the sea-level. The mercury column in the barometer sinks, as it is carried up in the atmosphere. This was first verified at the instance of Pascal by Perrior in 1548, when he carried a barometer up the heights of the Puy-de-Dome There can be established a relation between the amount of the fall of the mercury column and the height ascended.

It has been noted before in art 157 that the pressure of the air increases downwards to the surface of the earth just as pressure increases with the depth of a liquid. But in a liquid which is almost incompressible, pressure increases in proportion to the depth ; in air, however, it is not like that. As air is very compressible, a layer in the lower portion of the atmosphere is compressed by the weight of the superincumbent layers ; hence pressure in the air increases much more rapidly than in proportion to the depth. So the law for the variation of the barometric height with the altitude is not a simple one. Very complete tables have, however, been prepared by which the difference in height between any two places may be readily ascertained, if the corresponding heights of the barometer be known. For small elevations we may roughly take that the barometric height falls *1 inch for the first 900 ft, ascended* (or 1 mm. for 10 metres), 1 inch for the next 1000 ft., 1 inch for the next 1100 feet etc.

* An approximate calculation of the height of a mountain can be made in the following way :—

Let H be the height of the mountain

p_1, p_2 be the barometric readings in cms. at the top and bottom respectively.

t_1, t_2 be the corresponding temperatures.

$\therefore$ Pressure of a mercury column of height $p_1 - p_2$
$= (p_1 - p_2) \times 13{\cdot}59$ gms.

Evidently, this must be the weight of a vertical column of air, 1 sq. cm. in section and of height H. Let us assume that the

$$\text{average pressure of this air column} = \frac{p_1 + p_2}{2} = p, \text{ say}$$

$$\text{and} \quad \text{,,} \quad \text{temperature} \quad \text{,,} \quad \text{,,} \quad = \frac{t_1 + t_2}{2} = t, \text{ say}$$

Then we have H c.c. of air of a mean pressure p_1 and a mean temperature t. Its volume at N. T. P. is given by the relation

$$\frac{PV}{T} = \frac{P'V'}{T'} \qquad [Cf\ Heat,\ art.\ 41.$$

Here
$$\frac{p.H}{273+t} = \frac{760 \times V'}{273}$$

Or
$$V' = \frac{p.H}{273+t} \cdot \frac{273}{760}$$

$$\text{and wt. of this column} = \frac{p.H}{273+t} \times \frac{273}{760} \times {\cdot}001293 \text{ gms.,}$$

for the density of air at N. T. P. $= {\cdot}001293$ gms. per c. c.

$$\therefore \quad \frac{p.H}{273+t} \cdot \frac{273}{760} \times {\cdot}001293 = (p_1 - p_2) \times 13{\cdot}59$$

$$\text{whence} \qquad H = \frac{(p_1 - p_2) \times 13{\cdot}59\ (273+t)\ 760}{p \times 273 \times {\cdot}001293}$$

The same causes which make the pressure of air rapidly diminish as we ascend above the sea-level, produce a rapid increase in its pressure, when we descend into a deep mine.

165. Extent of Atmosphere.—As the molecules of air are continually tending to fly away from each other, it might be supposed that air would expand indefinitely into the open space beyond. But as the air expands, its temperature is lowered, which action is helped by the low temperature of the upper regions of the atmosphere, so that at a certain height the tendency of the air to expand decreases to such an extent that it is balanced by the action of gravity. It follows that the atmosphere is limited and exists like a cover round the earth.

But it is very difficult to tell exactly how far the air extends into the space. Were the atmosphere homogeneous *i.e.*, of the same density throughout, and of the same density as the air at the sea-level viz., o·oo129, its depth would have been some 8 kilometres or about 5 miles, for

$$\frac{13{\cdot}596\times76}{{\cdot}001293}=7{\cdot}988\times10^5\text{ cm},=7{\cdot}988\text{ km.}$$

$$=7{\cdot}988/1{\cdot}609\text{ miles}$$
$$=4{\cdot}97\text{ miles.}$$

The height of 5 miles computed on this imaginary assumption is called the Height of the **Homogeneous**

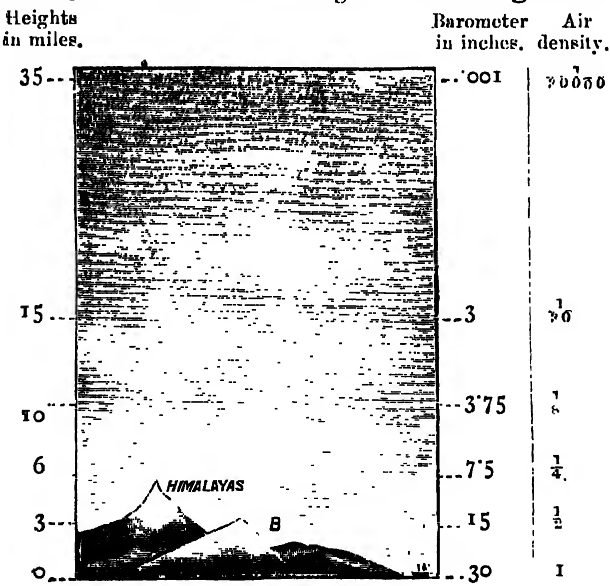


FIG. 186.

Extent and character of atmosphere.

Atmosphere. The tops of the Himalayas (H in fig 186) would have risen above it.

The highest point reached by balloons manned by aeronauts is 35,500 feet or about 7 miles. At this altitude the barometric height is only about 7 inches and the temperature is about 50°C. By sending up self-registering thermometers and barometers in unmanned balloons which burst at great altitudes, the instruments being protected by parachutes from the dangers of rapid fall, the atmosphere has been explored to a height of about 18 miles. The vast space above this is an unknown region.

"Fig. 186 shows, in the right-hand column, the densities of air at various heights in terms of its density at sea-level. In the next column are shown the corresponding barometer heights in inches, while the left hand column indicates heights in miles. At a height of 35 miles, the density is estimated to be but 1/30,000 of its value at sea-level. By calculating how far below the horizon the sea must be when the last traces of colour disappear from the sky, we find that at a height as great as 45 miles there must be air enough to reflect some light How far beyond this an extremely rarefied atmosphere may extend, no one knows. It has been estimated at all the way from 100 to 500 miles."

[A First Course in Physics by Millikan and Gale.

ARCHIMEDES' PRINCIPLE APPLIED TO GASES.

166. Buoyancy of the Air.—As air, like liquids exert pressure equally in all directions, Archimedes' Principle applies in this case too. In other words, bodies immersed in air or any other gas, are buoyed up, as in liquids, by a pressure equal to the weight of air or the gas displaced. The loss of weight of a body in air is demonstrated by means of the **Baroscope** shown in fig. 187. It consists of a scale beam, at one end of which a hollow glass sphere is supported, and at the other a lead counterpoise; the latter arm is also provided with a rider screw for the final adjustment.

Expt. 102. Adjust the rider screw until the beam is horizontal. Place the whole under the receiver of an air-pump and exhaust the air inside. The sphere sinks showing that its weight has apparently increased (fig. 187). Restore the former state to the beam by re-admitting the air.

When in air, the sphere as well as its counterpoise, is buoyed up by the weight of the air displaced. But as the sphere has a large volume, it displaces a larger volume of air and is consequently acted upon by a large buoyant force in air. Though the two are balancing each other in air, it is evident that the true weight of the sphere is greater than that of the counterpoise. On removing the air from inside the apparatus, this fact is proved.

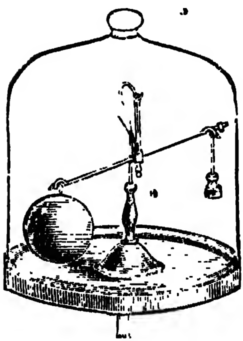


Fig. 187.
Baroscope.

It is thus evident that when a body is weighed in air, the weight obtained may be called its *apparent weight* and is less than its true weight in vacuo. In determining the weight of an object precisely, a correction for the buoyancy in air must be allowed for both the body to be weighed and the weights used.

It follows from the Principle of Archimedes that if the weight of a body is less than that of the air displaced by it, in other words,—if the body be lighter than air,—the body will be buoyed up and will rise in the atmosphere until it reaches a layer of the same density as its own; the force causing the ascent being the excess of the buoyancy over the weight of the body. This is the reason why smoke, vapour, a fire-balloon and air balloons rise in the air.

167. Balloons.—The buoyancy of the air is utilized for a most important purpose *viz.*, in the ascent of a balloon in air. A **Fire-Balloon** consists of a paper envelope with a wide opening below, in the centre of

which is a piece of sponge or cotton wick, held in a wire-frame, and soaked in mythelated spirits. The sponge is ignited and the bag is filled with heated air which, being lighter than the cold air, causes the balloon to rise.

A **Balloon** is essentially an air-tight envelope or bag of silk or some othet light material filled with some gaseous substance like hydrogen, coal-gas etc., which is lighter, bulk for bulk than the air at, the surface, of the earth and serves to float the apparatus in air (fig. 188). In the usual form it is spherical with a light car or basket suspended below it to carry passengers. The car is suspended by cords attached to a net-work covering the upper half of the balloon (fig 188). The necessary condition of the ascent is that the weight of the air displaced must be greater than that of the balloon and its load.

The difference between the weight of a balloon and that of the air displaced by it, is called the **Lifting Power** of the balloon. The three modes of inflating the balloon may be compared thus : the approximate weight of 1 cubic metre of air is 1300 gms : of hydrogen, 89 gms ; of coal gas, 750 gms ; of air heated to 200°C, 750 gms. So the lifting-power per cubic metre of hydrogen is .211 gms ; of coal-gas, 500 gms ; of heated air, 500 gms. Though coal-gas has a lifting-power much smaller than hydrogen, yet it is now generally employed on account of its cheapness, and of the facility with which it can be procured.

Balloons are not fully inflated at the commencement of the ascent, for as the ballaon rises, the density of the air diminishes and the external atmospheric pressure, on the balloon continually diminishes. In consequence the gas inside it expands in the same ratio as the pressure diminishes outside (see Boyle's Law) till the balloon is fully distended. Up to this time the lifting power remains nearly constant. Suppose for instance, the atmospheric pressure has diminished to one-half, the volume of the balloon will then be doubled ; it will then displace a volume of air twice as great as before,

but only half the density, so that the buoyancy will remain the same. This conclusion, however, is not quite exact, as the solid parts of the balloon do not expand like a gas.

When once the balloon is fully distended, if it continues to rise, its lifting-power diminishes rapidly, for the volume of the displaced air remains the same, but its density diminishes. A time, however, arrives when the weight of the air displaced is equal to that of the balloon itself; the balloon can rise no more and comes to rest in the region after a few oscillations, only to be drafted by the current in the air.

At the top of the balloon there is an opening, closed by a valve held to a spring, which the aeronaut can open at pleasure by means of a cord. When the aeronaut wishes to descend, he opens this valve by means of the cord, thus allowing the gas to escape. To rise again or

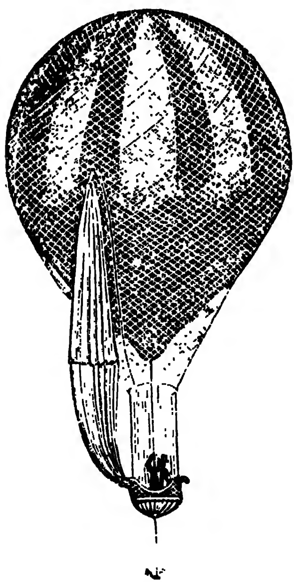


FIG. 188.

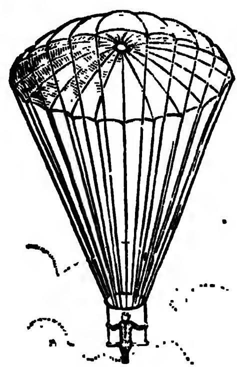


FIG. 189.

to make the descent less rapid, sand-bags kept as ballast in the car are gradually emptied.

The descent from a balloon in mid-air is sometimes effected by means of a **Parachute** (fig. 189). It is a huge umbrella like contrivance, from the circumference of which hang cords supporting a small car. The resistance of the air opens the parachute and acting up on its large surface moderates the rate of descent. It has a hole in the top, which allows that air to escape slowly and thus keeps the parachute upright.

The earliest recorded ascent of a balloon is credited to the Chinese on the occassion of the coronation of the Emperor To-kien at Pekin in the year 1306. The historical record after this was very meagre up to the latter part of the 18th century, when the brothers Stephen and Joseph Montgolfier started the beginning of practical aeronoutics. In 1783 they sent up a great paper balloon filled with hot air, which rose to a height of 1000 ft. but soon came to the earth again on the cooling of the hot air inside in the higher regions of the atmosphere. In their many subsequent experiments they attached a car to carry passengers in the fashion that has since become so familiar. About the same time M. Charles, a French scientist, was the first to ascend by means of a balloon which was filled with hydrogen. The balloon was sent up from Champ-de-Mars on August 29, 1783 amidst the booming of cannons and in the presence of 300,000 spectators who assembled despite a heavy rain. The balloon burst from the expansion of the gas in the higher and rarer atmosphere,—no allowance having been made for this unforeseen result. The same year witnessed the first crossing of the English channel by Blanchard, another French balloonist.

Montgolfier

Charles

Blanchard

Since then balloons multiplied rapidly and the ascents were too numerous to be recorded here. In 1804, Gay Lussac, a French physicist made an important ascent to a height of 23,000 feet. He found the air at that height to be extremely cold (at about $-9°C$); and the barometer fell to 12·6 inches The respiration and the circulation of the blood were accelerated in consequence of the diminished density of the air. The atmosphere at that height is extremely dry, pieces of paper becoming dried and crumpled as if they had been placed near the fire; the sky was nearly black and an absolute silence prevailed.

Gay Lussac.

Another balloon ascent which was remarkable, was that of the two daring English aeronauts, Glashier and Coxwell They claimed a record height of 37,000 feet in their ascension in September 5, 1862. At this

Glashier & Coxwell

height the barometer was noticed to stand about 7 inches. Such was the intense cold that Mr. Glashier fainted. For several discrepancies in their in observations and on the ground that their instruments were not of the highest reliability it is estimated that they could not have reached higher than 31,000 ft The world's height record rests with Professor Berson, a French aeronaut who on July 31, 1901 reached 35,500 feet without injury, as he carried oxygen with him for artificial inhalation.

Berson.

Almost from the very begining of ballooning, some method of directing the balloon to a pre-determined goal had been sought by the inventors, for one of the strong ambitions of the human race is to fly and not simply to rise in the air, though the latter is undoubtedly a necessary step. To accomplish this, a machine propeller is needed to drive the machine after it has been lifted up in the air, and a suitable rudder to direct its course. Such machinery will again have weight and the gas-bag must be enlarged to counterbalance it. The whole constitutes the **Dirigible Balloon** or the **Air-ship**. To reduce the resistance to motion, offered by the air, to the least possible amount, the globular form of the early balloon has been variously modified. Modern Air-ships, have gas bags of elongated cigar like shapes, and are propelled by gasoline engines, delivering a maximum of power with a minimum of weight. Some of the important ones of this type are Santos-Dumont, Zeppelin, Clement-Bayard, Wellman etc.

Airships.

168. The Flying Machines.—The term **Flying Machine** is applied to all forms of air-craft which are heavier than air, and which lift and sustain themselves in the air by mechanical means. In this respect they are distinguished from balloons which are lifted and sustained in the air by the lighter-than-air gas they contain.

Aeroplanes are those forms of flying machines which depend for their support in the air upon the spread of surfaces which are variously called *sails* or *planes*. They are commonly driven by propellers actuated by motors. When not driven by power, they are called **Gliders**.

Aeroplanes exist in several types—

1. The **Monoplanes,**—with one spread of surface. *e.g.*, the Bleriot, Santos-Dumond, Moisant etc.

2. The **Biplanes,**—with two spreads, one above the other, *e.g.*, the Wright, the Voisin, the Curtiss etc.

3. The **Triplanes,**—and *Multiplanes.*

Helicopters are machines which are lifted vertically and sustained in the air by propellers revolving in a horizontal plane and distinguished from the propellers of the aeroplane, which revolve in vertical planes.

The supporting planes of an aeroplane are on the topside, bent from front to rear, with a convexity of surface from upwards : the lifting power is believed to be in this curvature of the top-side. When driven in either direction, it leaves the air with a *downward* trend and is acted upon in its turn by an upward reaction which raises the machine.

The planes are helpful in maintaining the lift, while the blades of the propeller are turned by a motor to make the machine fly. These are possible only because air possesses elasticity and inertia. To stay up in the air, an aeroplane must move swiftly through it ; the heavier it is, the faster it must go. The common sport of 'skipping stones' explains why the machine, while gliding, does not fall ; a flat stone when thrown at a great speed in such a way that its flat surface touches the water, it continues skipping again and again until its speed is so reduced that the water surface where it touches last, has the time to give in ; and the weight of the stone carries it down. Wilbur Wright, when asked what kept his machine up in the air,—why it did not fall to the ground, replied, "It stays up because it doesn't have time to fall."

In practical aviation the density of the air is a controlling factor. An aeroplane meant to fly at a high altitude where the density of air is necessarily small, (for example, at an altitude of 5 miles where the density of air is [illegible] of that near the sea-level), it must either have huge supporting planes or an arrangement for a very high speed.

The air-ships and the aeroplanes are now-a-days largely used for military purposes ; the military experts assert that the dirigible is outclassed for warfare by aeroplanes which can operate in wind in which the dirigibles dare not venture.

Exercise —XIX.

1. State in a general way the use of the barometer

(*a*) in measuring the heights of mountains

(*b*) in indicating the state of the weather.

2. A solid floats partly submerged in a liquid when the vessel which contains it is in the air; if the vessel be placed in a vacuum, will the solid sink, rise or remain stationary ?

3. Why does the water gurgle out instead of issuing in a steady stream, when a bottle full of water is inverted ?

4. In selling diamonds by weight which of the following is advantageous to the seller.—

(*a*) the barometer should be high or low ?

(*b*) the weights used should be of rock-crystal or of platinum ?

5. 1 litre of hydrogen and a litre of air weigh about 0·09 gramme and 1·3 grammes respectively at a certain temperature (t) and pressure (p). What will be the capacity of a balloon weighing 10 kilogrammes, which just floats when filled with hydrogen having the same pressure (p) and the same temperature (t) as the air ? [*C. U.—1912*.

6. Describe an experiment to prove that air exerts pressure. How is this pressure measured ?

If a certain pressure is equal to that exerted by a column of mercury of height 760 mm, find its magnitude. (Density of mercury—13·6). [*C. U.—1917*.

7. Explain clearly what you understand by atmospheric pressure.

Describe experiments to prove the existence of atmospheric pressure. How is it determined ? If it is equal to that of 32 inches of mercury, find its magnitude. [Density of mercury =13·6.] [*C. U.—1918*.

8. Explain fully the meaning of the statement 'The atmosphere exerts a pressure of 15 lbs. per sq. inch, nearly.'

How would you verify the statement experimentally ?

[*C. U.—1919*.

9. Describe any form of barometer you have used in your laboratory. Give the directions necessary for reading the atmospheric pressure. [*C. U.—1921*.

10. A glass tube, 20 inches long, closed at one end and entirely filled with mercury, is inverted over a mercury trough. State what happens, giving reasons. [*C. U.—1922 ; '23.*

11. Describe an experiment to show that the principle of Archimedes can be applied to gases also. A flask is first weighed with its mouth open, then with its mouth well-corked by a rubber stopper ; what difference will you notice ?

[*Fat. U.—1919*

12. Describe with a neat sketch and an index of parts a good barometer, mentioning the precautions necessary to have a good vacuum on the top of the mercury ? [*Fat. U.—1919.*

CHAPTER XX.

BOYLE'S LAW.

169. Expansion of Gases.—The three variables in the case of a gas are its volume, pressure and temperature. To specify a quantity of a gas by volume, the temperature which it possesses and the pressure to which it is subjected, must be mentioned.

A gas, unlike a solid or liquid, alters considerably in volume for small changes of pressure, even though the temperature remains constant. The relation between the pressure and volume of a given mass of a gas *at constant temperature* is found to conform generally to a definite law, called **Boyle's Law**. The law was discovered in 1662 by the HON. ROBERT BOYLE (1626–1691) in England, and also independently, in 1676 by MARIOTTE in France.

Similarly the law that connects the rise of temperature and the increase of volume of a gas *under constant pressure* was first enunciated in 1787 by CHARLES, a Frenchman, and is called *Charles' Law*.

170. Boyle's Law.—The law states that *the volume of a given mass of gas at constant temperature is inversely proportional to its pressure.* Thus if p be the pressure and v be the volume of a given mass of gas; then, according to the law, we have

v varies as $1/p$

or $\quad v = k \cdot 1/p \qquad$ where k is a constant.

or $\quad pv = k \qquad \ldots \quad (1)$

Or again, if a fixed quantity of gas at contant temperature has volumes denoted by v_1, v_2, v_3, etc , under pressures denoted respectively by p_1, p_2, p_3,........., then we must have

$$p_1v_1 = p_2v_2 = p_3v_3 = \qquad \ldots \quad (2)$$

It will be seen that Boyle's law may be stated in terms of the pressure and density of the gas. For, if a given mass of a gas has a volume v_1 and density d_1 under a pressure p_1, and a volume v_2 and density d_2 under a pressure p_2 then since

$$\frac{d_2}{d_1} = \frac{v_1}{v_2} \quad \text{and} \quad \frac{v_1}{v_2} = \frac{p_2}{p_1}$$

we must have

$$\frac{d_1}{d_2} = \frac{p_1}{p_2} \qquad \ldots \quad (3)$$

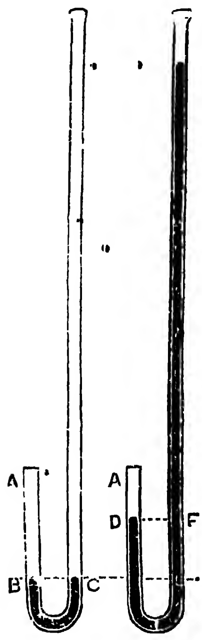
i. e., the density of a gas at constant temperature is directly proportional to its pressure.

Boyle's law may be experimentally verified by means of a tube shown in fig. 190 similar to what was used by Boyle in his experiment to establish the law, and is generally called *Boyle's Tube*. It is simply a glass U-tube having one arm shorter and closed at the top, about 6 in. long, and the other, a longer, open arm about 36 inches long and is mounted on a vertical board. Both the limbs of the tube are usually graduated in the same way from a zero mark at the same horizontal level.

Expt. 103. Pour a small quantity of clean mercury into a Boyle's Law tube, and adjust by tilting the tube so that the surface of the mercury in both the limbs is at the same level. Now the air enclosed in AB, the closed arm, is at the atmospheric pressure To get its volume, read the scale AB ; it is assumed here that the bore of the tube is uniform, and the unit in which the volume is measured is evidently the capacity of the tube per unit length.

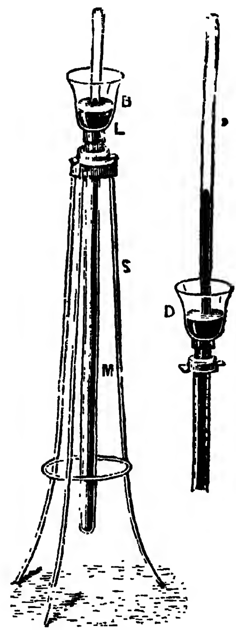
Now pour mercury again into the open limb ; notice that the volume of the air in the smaller limb is gradually reduced. Continue to do this until the volume of the contained air is half

of what it was at atmospheric pressure : in other words, AD in fig. 190 (B) is half of AB in fig. 190 (A). Measure the height of the mercury column above DF. It will be found to be exactly equal to the height of the barometer at the time of the experiment. The pressure at F is, therefore, equal to that of two atmospheres which must also be the pressure of the air in AD.



(A) (B)

Fig. 190.



(A) (B)

Fig. 191.

To verify Boyle's Law.

Thus the experiment proves that when the pressure of the air in the closed limb is *doubled*, the volume is *halved*.

Expt. 104. Arrange the apparatus as in the first part of Expt. 103 and note the volume of the air enclosed when it is at the atmospheric pressure.

Now pour mercury into the open limb, step by step, so as to raise the level about 3 inches at each step; and note the pressure and volume of the air in the closed limb at each step. The pressure at each step is given by the atmospheric pressure which is acting on the surface of mercury in the open limb, *plus* the pressure due to the difference of the mercury levels in the two limbs.

Arrange the readings in a tabular form.

It will be seen from these results that the value of the product of the volume of the air and the corresponding pressure is practically constant.

To demonstrate the truth of the law for pressures less than one atmosphere, a different apparatus, introduced by Regnault is required. It simply consists of a graduated tube and a deep trough to contain mercury, fixed on a suitable stand (fig. 191, A).

Expt. 105. Pour mercury into the graduated tube, until it is about two-thirds full, leaving the upper part to be occupied by the air. Place the thumb over the mouth of the tube, and invert it in a deep trough M, containing mercury (fig. 190. A) Lower the tube until the mercury inside and outside the tube is at the same level. Measure the length now occupied by the air which is at the atmospheric pressure.

Raise the tube slowly; the mercury recedes from the closed end, showing that the air in it expands, owing to decrease of pressure acting on it. Measure again the length occupied by the air, and determine its pressure by subtracting the height of the column of mercury standing in the tube from the barometric height observed from a Fortin's barometer. As before, the tube is assumed to be uniform in cross-section; hence the volume of the air in the tube is proportional to the length occupied by it.

Verify that PV is almost constant.

The apparatus now-a-days used in the laboratory for the above purpose is a much more convenient one than Boyle's tube. This consists of two glass tubes AB and CD connected by means of a long, thick-walled india-rubber pressure tubing, and fitted by means of adjustable clamps on two vertical uprights (fig 192). AB is uniform in cross-section and closed at the top and contains a certain quantity of *dry* air. The lower part of the glass tubes and the whole of the india rubber

tubing contain mercury. A wide scale runs along the vertical board in the middle. CD acts as a mercury reservoir The pressure to which the gas in the closed limb is subjected can be varied by raising or lowering the mercury reservoir, thus allowing the law to be tested for pressures *both* greater and less than the atmospheric pressure.

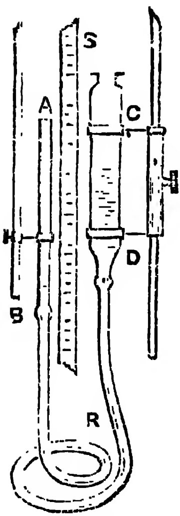


FIG. 192.

Expt. 106.* Fix the tube BA (fig. 192) about the middle of the stand. Slide CD until mercury stands at the same level within the two tubes. The pressure of the air enclosed is equal to the atmospheric pressure, and its volume is proportional to the length of AB occupied by the air. Read the barometer to get the atmospheric pressure.

Now raise the tube CD gradually; the volume of the enclosed air is reduced more and more The pressure at each step is equal to the atmospheric pressure *plus* the pressure due to a column of mercury equal to the difference of the heights of mercury in the two tubes. Note each time the volume of the air and the corresponding pressure.

In the *next* series of operations lower CD below AB. The pressure on the enclosed air is this time less than the atmospheric pressure by the difference of the heights of mercury in the two tubes. Take readings in several steps.

Find the product of P and V and show that this is almost constant.

The relation between P and V may be conveniently expressed graphically: if a curve be plotted so that the abscissae represent the pressures and the ordinates, the corresponding volumes of a given mass of gas at a constant temperature, the form of the curve is shown in Fig. 193. The curve is of the form of what is called the Rectangular Hyperbola.

* See Practical Physics by the Author.

More exact experiments performed by Regnault and others have shown that Boyle's law is not obeyed by gases at very

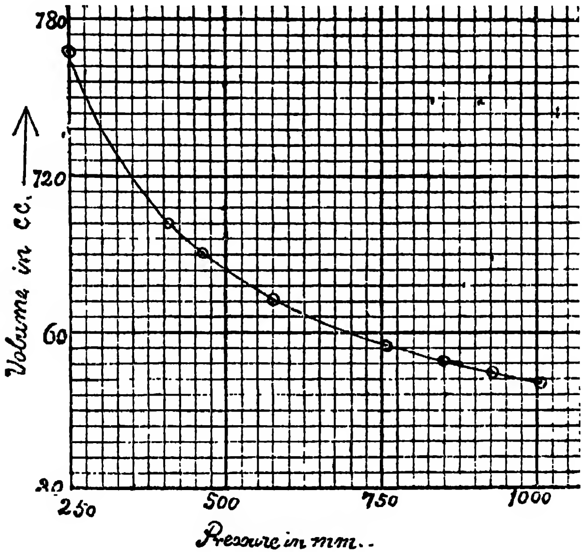


FIG. 193.

Variation of the volume of a gas with pressure.

high pressures. For ordinary pressures, however, the law holds very nearly in the cases of so-called permanent gases viz., oxygen, hydrogen, nitrogen, etc.

171. Manometer.—A Manometer or a pressure-gauge is an instrument for measuring the pressure of a gas of a vapour. The pressure is generally expressed in terms of atmospheric pressure and is often measured by means of a column of mercury.

A Siphon Gauge consists of a glass tube bent to the form of a U, as ABD in fig. 194 and contains mercury at the bend. The end D is in communication with the vessel, the pressure in which is desired to be measured.

When pressures which are not very considerable are to be measured, the end A of the gauge is often *open* to the air. Suppose that on connection with the vessel, the mercury levels on the two limbs are at C and D. Now the pressure at D equals the pressure at C (which is here the atmospheric pressure) *plus* the pressure of a column of mercury, the height of which is given by the difference of readings of C and D taken with the scale attached.

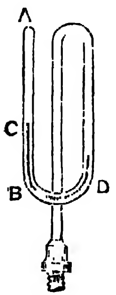


FIG. 194.
A Siphon-gauge.

The choice of a liquid to be used in the gauge will depend to some extent on the pressure to be measured; with a dense liquid like mercury, the 'head' of mercury *i.e.*, the difference of two levels in the two limbs would be small. If a liquid of smaller specific gravity be used, the 'head' necessary to measure a given pressure will be large in the inverse ratio of the specific gravities. Sulphuric acid (sp gr. 1·84 about) is often used; but it absorbs water quickly and has its density altered thereby. Water may be employed in some cases but it evaporates rapidly; further, the pressure due to the water vapour may cause error.

Expt. 107. Measure the pressure of the gas supplied in the gas-pipes of the laboratory.

For the measurement of *high* pressures, the end A of the gauge is closed as in fig. 194, so as to enclose a quantity of air. As the pressure on the mercury on the side D is increased, the mercury level in this limb is driven down, and the air in AC is compressed. The extent to which the air is thus compressed, indicates the pressure to which it is exposed and this pressure is obtained by the application of Boyle's law. The pressure at T is chiefly due to the compressed air; in most cases, the pressure due to column of the mercury of the height given by the difference of the levels of mercury in the two limbs, is negligibly small compared with that due to the compressed air.

For measuring *low* pressures, the tube AB contains no air and is completely filled with mercury. So long

as the end D is open, the atmospheric pressure forces the mercury up the arm AB to the top of the tube. When the pressure at D is sufficiently lowered, the mercury in AB falls. Suppose the levels are at C and D as in fig. 194; the pressure above C is zero; draw DE horizontal through D. Pressure at D equals that at E which again is measured by the height of the column EC. This form of gauge is commonly used with an air pump.

Exercise.—XX.

1. The volume of an air bubble increases six-fold in rising from the bottom of a lake. Find the depth of the lake (The barometer reading = 70 cms; and sp. gr. of mercury = 13.6).

2. An U-tube open at one end and closed at the other, is partially filled with mercury (density = 13.6). The closed end of the tube contains some air, and the mercury in the open limb stands 30 cms. higher than it does in the closed limb. Find in C. G. S. units the intensity of pressure on the air in the closed end of the tube. [*C. U.—1910.*

3. State Boyle's law and describe experiments made to verify it.

A faulty barometer contains some air which occupies 10 c.c. If it stand at 740 mm, when a true barometer indicates a pressure of 750 mm., find the volume, the air will occupy at the standard pressure 760 mm. [*C. U.—1911.*

4. State Boyle's law. Describe a method of verifying it experimentally.

What volume does a gramme of hydrogen occupy at 0°C., when the height of the mercurial barometer is 750 in millimeters? [1 c.c. of hydrogen weighs 0.0000958 grammes at 0°C and 760 millimeters.] [*C. U.—1913.*

5. A litre of air weighs 1.293 grammes at a pressure of 76 cm. and temp. 0°C. What will be the weight of a litre of air

at the same temperature, when the barometer stands at 78 cms? [C. U.—1915.

6 A given quantity of gas is allowed to expand to 1·5 times its original volume. What will be the pressure it will exert, if it were originally at a pressure of 750 millimeters of mercury, the temperature remaining constant throughout?

Describe an experimental arrangement by which your result may be verified. [C. U.—1916.

7. A volume of air at standard temperature and pressure is compressed to $\frac{1}{6}$th of its original volume. What will be the new pressure?

Describe an experimental arrangement which will enable you to verify the result. [C. U.—1920.

8. State Boyle's Law. Describe an experiment you would perform for verifying the law.

[C.U. - 1921.

9. State what happens in the following cases, giving reasons :

(a) A glass tube 20 inches long, closed at one end and entirely filled with mercury, is inverted over a mercury trough.

(b) A narrow glass tube open at both ends, is partially dipped in a vessel containing water. The upper end is closed by the thumb and the tube taken out of water [C. U.—1922.

10. State Boyle's Law. How may it be experimentally verified for pressures greater than the atmospheric pressure?

An accurate barometer reads 30 in. when one containing air above the mercury reads 24 in. If the tube of the latter be raised 3 in., the reading becomes 25 in. Find what length of the tube the air would occupy if brought to atmospheric pressure.

[C. U.—1924.

11. What is the pressure of a gas in a closed space due to it? Explain how the relation between the volume of a given quantity of gas and pressure may be determined experimentally. [Pat. U.—1918.

CHAPTER XXI.

HYDROSTATIC MACHINES.

172. The Pipette.—The instrument is shown in fig. 195. It consists of a glass tube with a bulb blown on to it about half way down. It is open at both ends and terminates below in a small tapering mouth. It is used for removing a liquid from one vessel to another. If water be introduced in the tube either by suction or by direct immersion in water, and if the upper end be closed with the finger, water will not fall being acted upon by the atmospheric pressure. The lower end is then placed in a vessel to which the liquid is to be transferred. On re admitting the air from above, the flow will begin and can be again stopped at pleasure.

Fig. 195. Pipette.

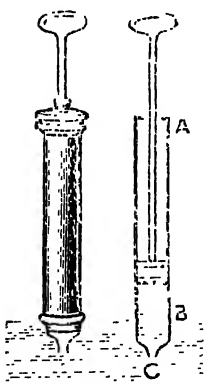


Fig. 196. Syringe.

173. The Syringe.—This instrument is the simplest form of the pump for raising water. It consists of a hollow cylinder AB (fig. 196), whose lower end is a nozzle C. There works within the cylinder a solid, air-tight piston.

Expt. 108. Place the nozzle C, of the syringe, with the piston

at the bottom of AB under the surface of water. Raise the piston ; the pressure of the air, acting on the upper surface of of the liquid, forces it into the cylinder to fill a vacuum which would otherwise be formed below the piston. Take out the syringe when sufficient liquid has been drawn up. The liquid may be ejected again through the nozzle C by reversing the motion of the piston.

The principle of the syringe and the various forms of pumps is that of suction This consists in enlarging the volume of a space to which the liquid has access ; the pressure within the space is thus reduced and the atmospheric pressure forces the liquid into the space to fill up the partial vacuum. This principle was not understood by the ancient philosophers who tried to explain the rise of the liquid by saying that *Nature abhors a Vacuum* (see art. 158). In inhalation, the muscles of the chest cause the lungs to expand, thereby reducing the internal pressure and the air is driven in. The act of drinking water is similarly explained.

Suction.

174. **Valves.**—Valves are used in most of the hydrostatic machines. They are made so as to yield to an excess of pressure on one side only ; an excess of pressure on the opposite side will close the valves. They thus allow passage to water, air etc., through the holes they close, in one direction but not in the other.

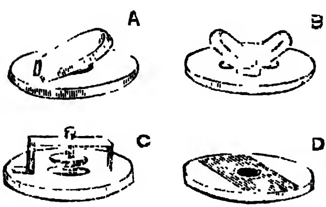


Fig. 197.

Types of Valves.

Fig. 197 (A) represents a *hanging flap* valve ; it is a flat disc turning about a hinge in its upper edge and thus opening or closing the passage over which it is fitted. In the ordinary bellows (fig. 198) the valve is a leather flap. This is raised when the bellows are being expanded, and allows the air to enter ; when the bellows are compressed,

the flap is pressed down tightly on the hole which is thus closed, and the air is forced through the nozzle.

Fig. 197 B represents a double flap valve, and fig. 197 C, a *conical* valve. When a fluid is forced through

FIG. 198.

The ordinary bellows.

the valve from a downward direction, the cone is raised and the fluid passes upwards. Pressure in the opposite direction serves only to drive the cone more closely against the hole over which it is fitted. The cone is prevented from moving far from the orifice by a suitable guide.

A form of valve used in many air-pumps is shown in fig. 197 D. It consists of a strip of oiled silk, secured firmly at both ends to a plate of brass over a narrow slit in the plate, meant for the passage of air. When air is forced against the valve through the orifice, the silk is lifted slightly and the air escapes; if air is forced in the other direction, the silk is pressed tightly down over the slit which is thus closed.

Theoritically, a valve should work whenever there is an excess, however small, of pressure on one side: in practice, however, no valve satisfies this condition; a definite excess of pressure is required before the valve will lift, and there is always some leakage.

175. The Common Pump.—The **Common Pump**, also called the **Suction Pump** consists of a barrel or cylinder AB (fig. 199), in which a piston P works smoothly and tightly: a long pipe BD is connected to the barrel at B and terminates beneath the surface of water which is to be raised. There are two valves, both opening upwards, one at V within the piston, closing an opening in it, and another at C the junction of the barrel and the pipe. The top of the barrel is generally provided

with a spout E. In the case of a hand-pump, the piston rod is worked by means of a lever, often a bent one, called the pump-handle.

To explain the action, let us start with the piston at the bottom of the barrel and the tube full of air above the water surface. As the piston is raised (fig. 199), the space below the piston is increased causing a fall of pressure in it : the atmospheric pressure acting on the

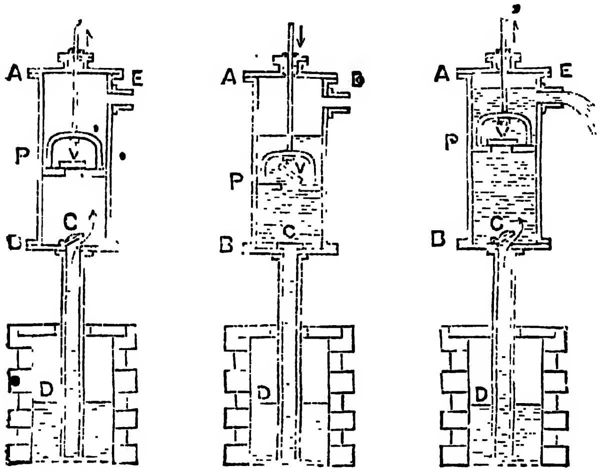


FIG. 199. FIG. 200. FIG. 201.

The action of an air pump.

valve V closes it ; the valve C, however, opens due to pressure of air in pipe, which is greater than that of the air above the valve, and the air in BD expands into the part of the barrel below the piston. This causes the pressure on the surface of water in the tube to be less than the atmospheric pressure which acting upon the water outside D forces it up in the pipe.

When the piston reaches the top, its motion is reversed. The pressure in the cylinder is increased, and the valve C is in consequence closed. When the

air below the pistons is got compressed to atmospheric pressure, it begins to escape by pushing the valve V upwards (fig. 200). This continues till the piston is again at the bottom of the cylinder.

Other complete strokes follow, the water rising higher and higher to the cylinder until it begins to collect in the cylinder. When the piston is again lowered, water is forced through the valve *v* and at the next up stroke of the piston flows out by the spout E (fig. 201).

Since the water is raised in the tube solely by the atmospheric pressure, it follows that *the height of the piston above the surface of the water must never exceed the height of a water barometer*, (*i e*,—about 34 ft.). In practice, taking into account the weight of the valve etc., the limit of the working height of the piston is less than 34 ft.

176. The Force Pump.—This differs from the common pump in as much as the piston P is solid and has no valve : a pipe DE rises from close to the bottom

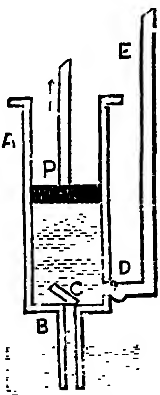


FIG. 202.

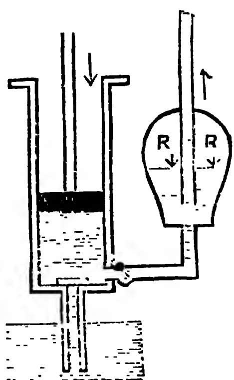


FIG. 203.

of the cylinder and is closed with a valve D (fig. 202) opening outward from the cylinder.

At each down-stroke the water collected in the barrel is forced out through the valve D and up the delivery pipe E, and at each upstroke the valve D closes due to the back-pressure of the water in the pipe E and water collects within the barrel (fig. 203).

The height to which the water can be forced depends on the force applied at the handle and the strength of the pump.

The drainage of deep mines is usually effected by a series of pumps. The water is first raised by one pump to a reservoir into which dips the suction tube of a second pump which sends the water up to a second reservoir, and so on. The piston rods of the different pumps are all joined to a single rod called the *spear* which receives its motion from a steam engine.

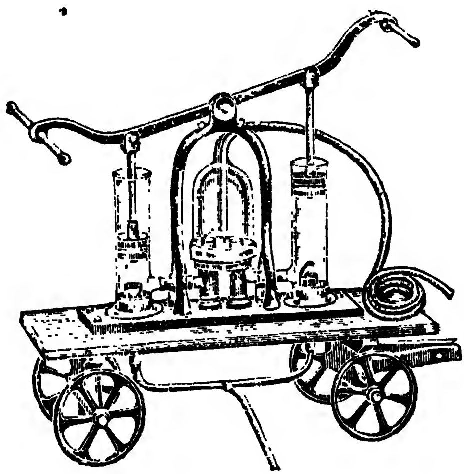


Fig. 204.
Manual Fire-engine.

The flow in the delivery tube of the force-pump, as just described. will be intermittent, the water flowing only during the down-stroke of the piston. To obtain a

continuous stream, two force-pumps may be so joined as to have a common delivery tube and the two pistons in the two barrels worked by a common handle, so that as one piston descends, the other ascends. Even then the action of the pistons momentarily stops, when their motions are reversed.

In order to produce a continuous jet of water from the hose, an **air chamber** is provided with the pump. This is simply a large metal dome (R, in fig. 205) partly filled with air. The delivery tube leads into this chamber whence a hose E, whose end is well below the air in the chamber, leads up to the height required When the water is rapidly pumped into the chamber, it rises above the lower open end of the hose, and compresses the

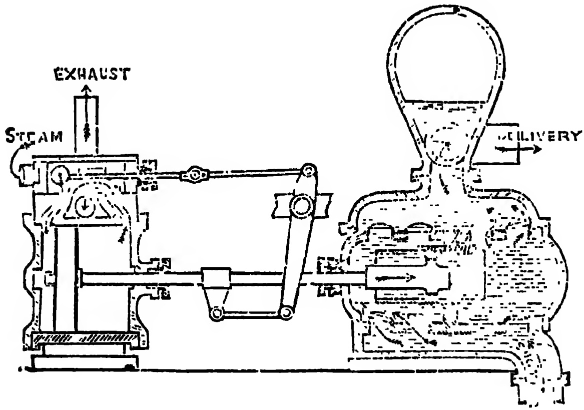


FIG. 204.
Steam Fire-engine.*

air in the chamber, while part of the water is forced out at the same time. During the upward stroke of the piston the valve T is closed and the air in the chamber being no longer subjected to the pressure expands, thus

* The figure is adapted from a drawing given in the First Course of Physics by Millikan and Gale.

driving the water up the hose. Thus a continuous flow of water is maintained.

The **Manual Fire-engine** (fig. 204) consists of two force-pumps connected to a common air-chamber. The **Steam Fire-engine** (fig. 205) is a double-action force-pump with a horizontal barrel. The piston is driven backwards and forwards by steam power, and water enters the barrel on the two sides of the piston, alternately. Each half forms a complete pump. The student will find it quite interesting to follow the action of the pump from a study of the diagram given.

177. The Siphon.—This is an instrument by means of which a vessel filled with a liquid may be emptied, when the ordinary process of pouring the liquid off is not convenient or desired. It is a bent tube CD (fig. 206) open at both ends, one limb being longer than

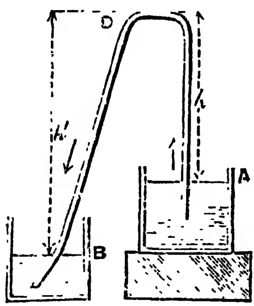


Fig. 206.

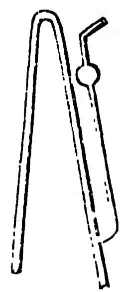


Fig. 207.

the other. It is first filled with the liquid to be drawn off; the ends are then temporarily closed with the fingers, and the shorter leg is placed below the level of the liquid in the vessel to be emptied. The other end is outside this vessel and below the level of the liquid surface. On opening the two ends, the liquid at once flows through the tube.

The principle of the action of the instrument is readily understood. Let CD be the highest portion of the tube, which is horizontal, and let h and h' denote the *vertical* heights of the points C and D respectively. Then

$$\text{Pressure at C} = \pi - hdg$$

$$\text{Pressure at D} = \pi - h'd'g$$

where π is atmospheric pressure and d the density of the liquid.

$$\text{Now} \quad h < h'$$

$$\therefore \text{Pressure at C} > \text{Pressure at D.}$$

Hence water will flow from C towards D, and the atmospheric pressure will again raise water to C : thus a continuous flow will be maintained.

The two conditions which must hold, so that the siphon can act are—

(1) The level A of the liquid in the vessel which is to be emptied, must be above the end B of the siphon tube by which the liquid will flow out.

(2) The height of the top of the siphon above the liquid in the vessel to be emptied must be less than the height of the corresponding liquid barometer.

For convenience in filling, the siphon is often made in the form, called **Aspirating Siphon**, shown in fig. 207 where it is provided with a side-tube. One end of the siphon is inserted in the liquid to be removed while the other end is closed, and the operator applies suction at the side-tube till the liquid flows over. In siphons for commercial purposes, the suction is usually produced by a pump.

The siphon may be employed to produce the intermittent flow of a liquid. Fig. 208 represents a cup in which there is a siphon with its shorter arm terminating near the bottom, while the longer arm passes through the bottom. If liquid be poured into the cup, the level will gradually rise both in the vessel and in the shorter branch of the siphon, till it reaches the top of the bend, when the tube is filled with the liquid and begins to discharge the liquid. If the liquid escapes by the siphon faster than it is supplied to the vessel,

the level in the cup gradually sinks until the shorter branch no longer dips in the liquid. The siphon is then empty and the flow ceases. If the supply of the liquid be allowed to continue, the siphon will recommence its action, when the level of the liquid again rises to the level of the bend.

In **Tantalus' Cup** (fig. 208) the siphon is concealed inside a figure of Tantalus, placed in the vase, whose mouth is just above the top of the siphon. Water is poured into the vessel, and no sooner does it reach the top of the siphon and approach his lips than the water all flows away through the siphon, leaving him as thirsty as ever.

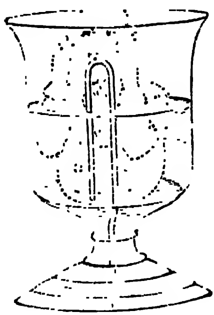


FIG. 208.

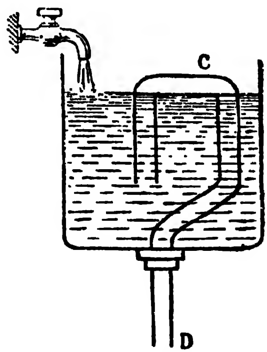


FIG. 209.

The siphon may be employed to produce the intermittent flow of a liquid. Fig. 209 represent a vessel in which there is a siphon with its shorter arm terminating near the bottom, while the longer arm passes through the bottom. If a small stream of water flows into the vessel, the level will gradually rise both in the vessel and in the shorter branch of the siphon, till it reaches the top of the bend, when the tube is filled with the liquid. The siphon then acts, and a powerful rush of water issues through the pipe S until the vessel is emptied. If the

supply of the liquid be allowed to continue, the siphon will re-commence its action when the level of the liquid again rises to the level of the bend.

The same principle is applied in the automatic *flushing tanks* used for sanitary purposes.

It is to an action of this kind that the *Natural Intermittent Springs* are generally attributed. Suppose a subterranean reservoir communicate with an outlet by a bent tube forming a siphon, and is fed by a stream of water at a slower rate than the siphon is able to discharge it. When the water reaches the top of the bend, the siphon is charged, and the reservoir is emptied. The flow will begin again when the siphon is charged after a time.

178. Mechanical Air Pumps.—The air-pump is an instrument constructed for the purpose of pumping air out of a closed vessel. Air pumps may be of two distinctly different types: one type being known as the *mechanical air-pump*, and the other as the *mercury pump*.

The air pump was invented by Otto von Guericke about 1650. It is almost identical in construction and similar in action with the common water-pump described in art. 175. This consists of a cylindrical metal barrel AB (fig. 210), in which an air-tight piston, P, can be worked up and down by a handle. This piston has a valve opening upwards. The barrel communicates through the tube D with the bell-jar receiver or a vessel R, to be exhausted, which fits air-tight on the flat, circular disc EF. At the junction of the barrel and the tube D, there is a second valve *a* also opening upwards. D is provided with a stopcock (not shown in the figure), by turning which air may be admitted within the receiver. To indicate the extent of exhaustion in R a manometer may be connected with the pipe D by a brass side-tube.

To understand the action, suppose the piston to be at the bottom of the barrel. In the up-stroke, the valve in the piston at once closes and the pressure of air within

the barrel below P falls. The air in the receiver and tube lifts the valve *a*, and expands into the barrel; thus the pressure in R is reduced. When the piston is forced downwards (fig 211), it compresses the air in the barrel below it. This at once closes the valve *a* and when the pressure of air in the barrel becomes greater

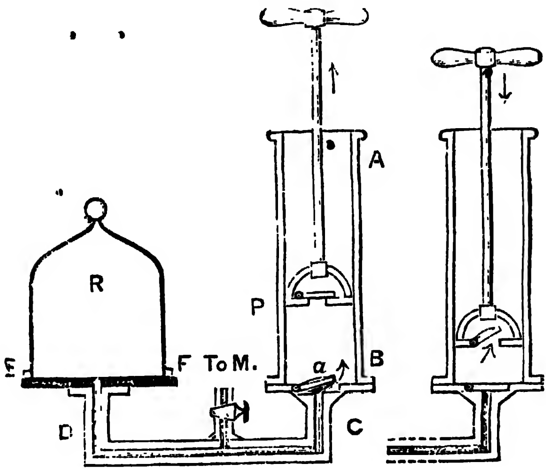


FIG. 210. FIG. 211.

An air pump in action.

than the atmospheric pressure, the piston valve opens and allows the air to escape from beneath the piston to the upper part of the barrel and thence to the outer air. At the next upstroke the air left in the receiver will again expand so as to fill both the receiver and the barrel. This process of exhaustion goes on until a fairly low vacuum is provided in the receiver.

To calculate the density of the air left in the receiver after a number of strokes is rather simple. Let

V = volume of the receiver R and pipe D.

v = volume of the barrel between the two valves when the piston is at the top of its upstroke,

It is clear that at each upstroke a volume v of air expands to $V+v$, and thereby becomes rarefied. If the original density of air be D, the density d_1 after the first upstroke is given by

$$VD = (V+v).\, d_1.$$

Or $$d_1 = \frac{V}{V+v}.\, D$$

Similarly, the density d_2 after the second upstroke,

$$d_2 = \frac{V}{V+v}.\, d_1 = \left(\frac{V}{V+v}\right)^2 . D$$

Thus the density of air left in the receiver after n upstrokes is given by

$$d_n = \left(\frac{V}{V+v}\right)^n . D$$

According to Boyle's Law, pressure of a gas is proportional to its density, we have

$$p_n = \left(\frac{V}{V+v}\right)^n . P$$

where p_n is the pressure after n strokes and P is original atmospheric pressure.

The result shows that the value of d_n can never become zero (indicating a perfect vacuum), but it may be made very small after a sufficient number of strokes, provided the pump is mechanically perfect.

But a pump of this pattern is never mechanically perfect. There is always a certain amount of leakage in action. Then there must always be a small *clearance i.e.*, a space left at the bottom of the barrel even when the piston is pushed 'full home.' After pumping for sometime there comes a stage when the valves do not open, and the air between them alternately expands into the barrel and is forced back into the clearance. Further, the valves, however light, require a force to be opened, and when the pressure of the air in the receiver becoms very low, it is unable to raise the valve *a* during the upstroke of the piston. Another point to be noticed in the working of the pump is that when the exhaustion is carried to a certain extent, the excess of pressure on the upperside of the piston over that of the air in the barrel below it makes the pump hard to work.

179. Double-barrelled Pump.—This is shown in fig. 212. In the ordinary single-barrelled pump no air is expelled in the downstroke ; in the **Double-barrelled Pump,** also called *Hawksbee's Air Pump*, there are

two barrels instead of one, and the pistons are worked up and down by means of a *rack-and-pinion* arrangement, so that when the pinion K is turned by a lever handle *m*, one piston rises as the other falls ; thus air is exhausted during each stroke. It will be noticed in fig. 212 that the passages from the two barrels unite into a single passage.

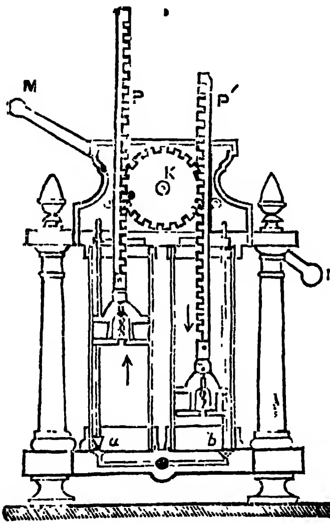


FIG. 212.
Hawksbee's air-pump.

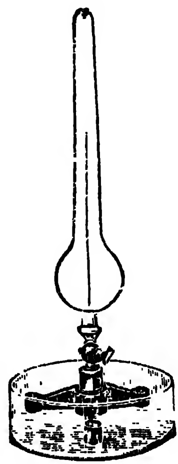


FIG. 213.
A fountain in vacuo.

This arrangement possesses two advantages. First, the air is exhausted twice as quickly as with a single barrel. Secondly, since the atmospheric pressure tends to depress each piston, its effect on one of the pistons which is rising, is just balanced by that on the other which is descending : less force, therefore, is necessary to raise the piston than in the ordinary pump.

But the second advantage exists rather at the beginning of the stroke only, for the air below the descending piston is

compressed, and its pressure increases till it becomes equal to that of the atmosphere and raises the piston valve. During the remainder of the stroke, the atmospheric pressure on the other piston is entirely unbalanced. As the exhaustion proceeds, the compensating action at all parts of the stroke becomes more complete ; the pump, accordingly, becomes easier to work.

USE OF THE AIR-PUMP ;—

At the time when the air-pump was invented, experiments were devised to demonstrate the effect of a vacuum, some of which have already been described, *e.g.*, the bursting of a bladder (fig. 171), the experiment with Magdeburg hemispheres (fig. 176), Mercury Rain (fig. 105), inflation of a toy balloon (fig. 169), Guinea and Feather experiment (fig. 47).

Fig. 213 represents a *Fountain in Vacuo.* It is simply an elongated flask, the base of which is pierced by a tube fitted with a stop-cock below and terminating in a fine nozzle within the flask.

Expt. 109. Screw the flask to the plate of an air-pump and exhaust the air from inside it. After closing the stop-cock transfer the flask in a vessel of water and open the stop-cock. The water being pressed by the atmosphere, is forced through the nozzle in a jet, as shown in the drawing.

Besides its use in the laboratory, the exhausting air-pump finds an application in many industries. It is employed in sugar refinery to lower the boiling point of the syrup : in exhausting the globes of incandescent electric lamps ; in parts of ice-making machinery : in exhausting the air from vessels meant for preserving foods etc., etc.

180. Condensing Pump.—This is an air-pump for compressing the air. It consists of a barrel AB, in which works a piston P (fig. 214). AB communicates at one end through a stop-cock with the Receiver or the vessel into which air is to be compressed. Both the piston and the end of the barrel contain valves, E and F, opening towards the receiver.

Let the piston be at the end of the barrel near the valve F. In the *backward* stroke, the pressure in the

barrel below the piston is reduced ; the valve F is closed by the pressure in the receiver while the atmospheric pressure opens the valve E, and the barrel is filled with air at the atmospheric pressure. In the *forward* stroke, the valve E is closed and F is opened ; hence all the air from the barrel is forced into the receiver. This process is repeated in every complete stroke of the piston.

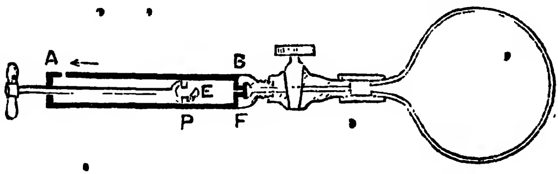


Fig. 214.
A condensing pump.

The piston valve is not necessary, if there be a hole in the side of the barrel just below the outermost position of the piston.

The pressure of the compressed air after a given number of strokes of the piston may be easily calculated. Let

V = volume of the receiver

v = volume of the barrel

D = density of the atmospheric air.

At each backward stroke, a volume v of air at atmospheric pressure enters the barrel. At each forward stroke this air enters the receiver.

Hence after n *complete* strokes the mass of air in the receiver $= (V + nv).\ D$

But its actual volume is V. Let its density be d_n.

Then $V.\ d_n. = (V + nv).\ D$

$\therefore \qquad d_n. = (1 + nv/V).\ D$

If Boyle's law is assumed to hold here, the pressure within R after n strokes is given by

$$f_n = \pi(1 + nv\ V)$$

where π is the atmospheric pressure.

In the common *Bicycle Pump* or *Cycle Tyre Inflator* (fig. 215), the valve in the piston is replaced by a con-

trivance called the *Cup-valve* ; A cup-shaped disc of leather, a little larger than the barrel of the pump is attached to a loosely fitting metal piston composed of two circular plates of smaller diameter than the barrel between which the leather is held. During the up-stroke

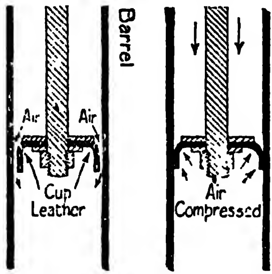


FIG. 215.

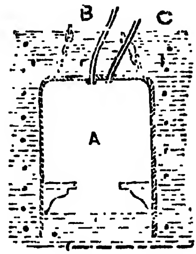


FIG. 216

the cup collapses inwards and allows air to pass by it : on the down-stroke the leather presses tight against the walls of the barrel preventing the escape of air round the piston.

181. Use of the Compressed Air.—Compression pumps are used in forcing carbonic acid gas into reservoirs containing the water which is to be aerated. Compressed air is largely used in supplying air to a *Diving-bell* or a *Caisson*. A **Diving Bell** (fig. 216) is an apparatus for enabling a man to descend to a considerable depth under water to examine the foundation of a pier etc. It is a heavy, bell-shaped vessel of iron, closed at the top and open at the bottom and contains a platform inside It is lowered by means of chains and sinks of its own weight. As the bell descends, the pressure of water increases and compresses the air in the interior. Hence, to prevent water from rising into the bell, and also to enable the workmen to breathe, a constant supply of air is pumped into the bell through a tube from the surface by means of a condensing pump, the surplus air bubbling out from the lower edges of the bell.

The so-called *Caisson*, much used in bridge building, is simply a stationary diving bell sunk to the bottom of the water and filled with compressed air at the same pressure as the water outside.

An **air-gun** may be described as a valveless condensing pump Fig. 217 is a section of a 'Gem' air-gun with the mechanism set ready for firing. In the stock of the gun, is the *cylinder*, in which an accurately fitting and hollow *piston* works. There is a stout steel *spring*, compressed between the inside end of the piston and the upper end of the butt. To set the gun, the *catch*, is pressed down so that its hooked end disengages from the stock, and then the *barrel*, is bent downwards on pivot P. This slides the lower end of the *compressing lever* towards the butt; and a projection on the guide B, working in a groove, takes the piston with it. When the spring has been fully

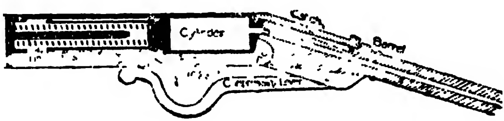


FIG. 217.

An air-gun.

compressed, the triangular tip of the *rocking cam*, R, engages with a groove in the piston's head and prevents recoil when the barrel is returned to its original position. On pulling the *trigger*, the piston is released and flies up the cylinder with great force, and the air in the cylinder is suddenly compressed and driven through the bore of the barrel, blocked by the leaden bullet, to which the whole energy of the expanded spring is transmitted through the elastic medium of the air. There are several other good types of air gun, all of which, however, work on the principle described above.

The action of *Pneumatic Tyres* in reducing vibration and increasing the speed of vehicle is explained by figs. 218 and 219. When the tyre encounters a large stone, it laps over it (fig. 218), and reduces the

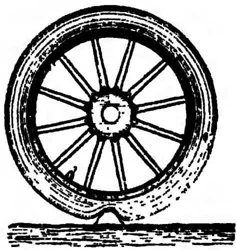


FIG. 218.

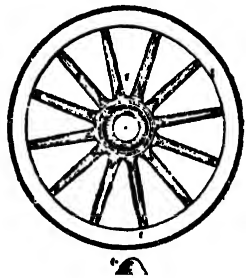


FIG. 219.

deflection of the direction of movement. When an iron-tired wheel (fig. 219), however meets a similar obstacle, it has to rise right over it, often jumping into the air, which necessarily causes a change in direction of motion : and this means a loss of forward velocity.

Compressed air is also used in manometers in the liquefaction of gasses, in driving excavating machines, in propelling torpedoes within water, in the air-brakes of a railway train, in driving pneumatic despatch tubes. In the pneumatic post, a carrier cylinder containing despatches moves air-tight in an underground smooth metal tube ; by exhausting the air at the forward end of the carrier and compressing the air at the other end, the train is blown through the tube with great velocity.

182. Hiero's Fountain—It derives its name from its inventor, Hiero, who lieved at Alexandria, 123 B. C. It consists of a dish A and two globes B and C. A tube D runs from A to the the bottom of the lower globe C (fig. 220), a second tube E connects the upper parts of the two globes, and a third tube F proceeding from the

bottom of the upper globe B passes through a cork in the centre of the dish A and ends in a fine jet.

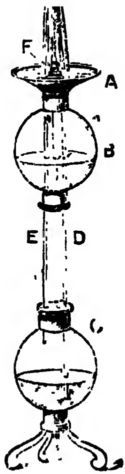


FIG. 220.
Hero's fountain.

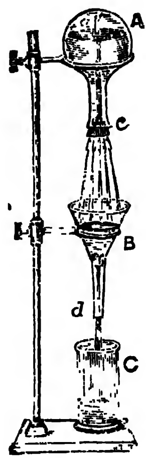


FIG. 221.
Intermittent fountain.

Expt. 110. Fill the upper globe with water by removing the cork in A. Replace the tube F and pour water in the dish. The water flows through D to collect into the globe C and displaces the air in it which passes up into the globe B and presses on the water there. Thus the water from B is forced out in a jet as represented in the figure.

183. Intermittent Fountain.—Fig. 221 shows the plan of working of an intermittent fountain. It consists of a globe A formed by a flask held in an inverted position, a basin or a funnel B and a collecting vessel C. The rubber stopper of the flask is pierced with holes through which run the efflux tubes *c* and a central tube open at both ends extends nearly to the inverted bottom of the flask above and to the middle of the funnel below. A piece of cork with a jet tube is fitted at the outlet of the funnel.

Suppose A is filled with water which will flow through the efflux tubes *c* and air will pass up through the central tube. But as the water issues from the efflux tubes much faster than it escapes from the funnel, the level of water in the funnel rises till the lower end of the central tube is covered. As the external air cannot now enter A, the pressure of air in A rapidly diminishes until the efflux from the tubes *c* is stopped by the atmospheric pressure. But as water continues to flow out of the funnel, the bottom of the central tube is again uncovered, air enters and the flow recommences; the same changes will then be repeated.

184. Pneumatic Inkstand.—It consists of a glass vessel A of the shape of that shown in fig. 222, communicating at the bottom with an open tubulure B. The inkstand is almost wholly filled with ink, care being taken to hold it in an inclined position, so that the air inside A may escape. While at rest, the level of the ink inside A is higher than that in the tubulure B, the pressure of the air inside being a little less than that of the atmosphere on the ink in the tubulure. With use, the level of ink in B sinks; as soon as it is lower than the corner C, a bubble of air passes into A. Now the pressure inside A is increased, and the level of ink descends in A and ascends in B. When the level of ink inside sinks to the level of C, the inkstand requires refilling. Such a vessel prevents ink from too rapid evaporation.

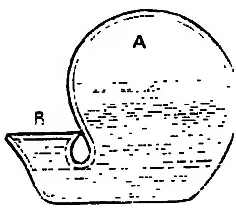


FIG. 222.
Syphon Ink-stand.

Exercise.—XXI

1. Explain the action of the siphon. Does it act in a vacuum ?

A small hole is made in one leg of a siphon, how does this affect its working ?

2. If the receiver of an exhaustion pump has double the volume of the barrel, find the density of the air remaining after 10 strokes, neglecting leakage etc.

3. Air is forced into a vessel by a compression pump whose barrel has 1/10 of the volume of the vessel. Compute the density of the air in the vessel after 20 strokes.

4. Describe in detail with a diagram a condensing pump and its mode of action.

The barrel and receiver of a condensing pump have capacities of 75 c.c. and 1,000 c.c. respectively. How many strokes will be required to raise the pressure of the air in the receiver from one to four atmospheres ? [*C. U.*—1925

5. Describe and sketch the Common Force-Pump.

CHAPTER XXII.

MOLECULAR MOTION IN GASES.

186. Molecular Motions in Gases.—The molecular motion in solids and liquids have already been dealt with in articles 118 and 151. Convincing evidence of the supposition that the molecules of a gas also are in a state of continual and rapid motion is found in the familiar observations of diffusion in gases, tendency of gases to indefinite expansion and the variation of gas pressure with alterations in its volume.

If ammonia, or chlorine, or a volatile perfume be brought into a room, the odour becomes perceptible in a very short time in all parts of the room, even though the air inside the room be still Again, it is shown that if two globes shown in fig. 223 provided with stop-cocks be filled, the upper one with hydrogen and the lower one with carbon dioxide gas and be screwed to one another, the chemical analysis after a few hours shows that in spite of the great difference in density,—for carbon dioxide gas is nearly 22 times heavier than hydrogen,—the two gases mix uniformly in the two globes. This passing of gases into one another, in apparent violation of the laws of weight is called the Diffusion of gases.

Diffusion of Gases.

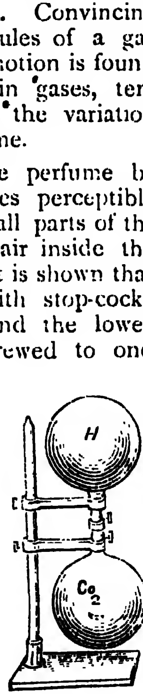


FIG. 223.

The most striking property that we notice in a gas is that of its unlimited expansibility. Experiments to illusttate this have been given in arts. 157 and 170. Again, the very simple

Expansibility of gases.

fact that a high degree of exhaustion may be brought about an air-pump depends upon the fact that the air in the receiver expands and fills up the entire vessel at each up-stroke of the piston. Indeed, but for this property of gases the pump would have been useless.

The facts can be satisfactorily explained by the only supposition that *molecules in a gas are in continual motion.* Thus, when the piston is drawn up, the molecules of air which are already moving in that direction, follow it up. Again as the piston can be worked very rapidly to bring about a rapid exhaustion, it follows that *the velocity of the molecules of a gas must be very considerable.*

It follows that gas molecules must, in course of motion, strike the boundary walls of the vessel which contains the gas and exert an outward pressure on them, the pressure being called the pressure of the gas.

Pressure of a gas.

Further, *the moving molecules must be perfectly elastic,* so that after each collision they rebound with the same velocity as before ; otherwise, their momentum would decrease with each collision and the pressure of a gas would decrease with time, which, however, is not the fact. Hence it is inferred that the *molecules are in a state of perpetual motion.*

186. The Kinetic Theory of Gases.—The assumptions mentioned above may be summarized into what is called the **Kinetic Theory of Gases.** It states that

The molecules of a gas are in a state of rapid perpetual motion in straight lines. The molecules are continually colliding against the walls of the boundary vessel and against one another. Further, they are perfectly elastic and rebound after a collision without any loss of momentum.

Assume that a closed vessel contains n molecules, and that the ceaseless cannonade of molecules on the walls of the vessel produces an average pressure p.

Imagine *n* similar molecules to be squeezed into the same vessel,—in other words, the density to be doubled,—the number of blows struck per second against a surface, and hence the pressure, are doubled. This is but another way of stating Boyle's Law.

Boyle's Law.

Except under very high pressures the molecules of a gas are at distances apart which are large in comparison with the diameter of the molecules, and intermolecular attraction is quite negligible; thus when steam is condensed to water, it shrinks to 1/1600 of its original volume, air liquefied is reduced to 1/800 of its ordinary volume. At high pressures molecules of some gases are too crowded to have this attractive force neglible. Then Boyle's law does not hold good.

Molecular attraction in gases.

The temperature of a gas is associated with the molecular kinetic energy. If heat be supplied to a quantity of a gas, the speed of the molecules increases. Now if the volume remains constant, the number of molecular impacts on the boundary walls, in other words the pressure of the gas, increases; if the pressure be constant, the volume increases.

Temperature of a gas.

At the same temperature, the average velocity of the molecules is not the same for all gases. At normal temperature and pressure 1 c.c. of air exerts a pressure of 1033 grams per sq. cm., and under the same conditions, 1 c c. of hydrogen exerts the same pressure. But the latter gas weighs only one fourteenth as much as the air. It is evident that to exert the same pressure the hydrogen molecules must be moving much more rapidly than the air molecules; the average velocity of the air molecules under normal conditions is calculated to be 450 metres per second, and that of the hydrogen molecules the enormous speed of about 1700 metres or 1 mile roughly per second.

Molecular velocity.

187. Diffusion of Gases through Porous Walls.—Further evidence of the difference in the molecular velocities of different gases is obtained in the phenomenon of diffusion of gases through porous vessels.

Expt. 111. Close the open end of a porous cylindrical cell of unglazed earthenware with a rubber stopper. Fix the end of a long, narrow glass tube of the shape shown in fig. 224 through the stopper; clamp the tube vertically. The lower part of the tube is bent into a U-form and contains coloured water.

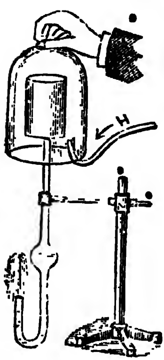


FIG. 224.

Place a gas jar containing hydrogen over the porous cylinder. The issue of a jet of water from the mouth of the tube shows that the pressure of the gas inside the cell is rapidly increasing. Remove the bell-jar; the hydrogen inside the cell now diffuses outwards. Water at once begins to rise in the tube, showing that the inside pressure is now rapidly decreasing.

The explanation is as follows:—If there are as many hydrogen molecules per cubic centimetre outside the cell as there are molecules per cubic centimetre inside, then, since the velocity of the latter is four times as great as that of the former, the hydrogen molecules will strike the partition four times as frequently as the air molecules strike it, and the former will pass inwards through the pores of the partition four times as rapidly as the latter. Since the inside is thus gaining molecules faster than it is losing them, the pressure in it increases. When the bell jar is removed, the hydrogen which has passed inside now begins to pass out faster than the outside air passes in, and hence the inside pressure is diminished.

ANSWERS.

Exercise II (Page 28)

2. (*b*) 1 ft. = 30·48 cm ;
   1 in = 2·54 cm.
4. 1 Kilogram = 2·2046 lbs.
   1 gram = ·002 lbs.
5. About 64 gms.

Exercise III (Page 46)

5. 8 min. 39·6 sec. : 120°
6. $\frac{11}{7}$ ft. per sec^2.
9. 5 ft. per sec ; 5·38 ft. per sec.
10. 4·24 mi. per hour.
11. 59·31 ft. per sec.

Exercise IV (Page 75)

3. (*a*) 18 gm ; (*b*) 20·78 ozs ; (*c*) 145 kg ; (*d*) 4·02 ozs.
4. 25 lbs. 5 40 lbs.
6. $7\frac{11}{16}$ lbs ; ·$237\frac{3}{8}$ lbs.
7. 3·46 lbs.
8. 3 min. 53·4 sec.
10. 3 ft from the man who bears 94 lbs.
11. 32 lbs.

Exercise V (Page 88)

1. 576 ft. ; 176 ft
2. $18\frac{1}{3}$ units. 3. 350 ft.
4. 0·16 ft. ; 4·4 ft.
5. 2·003 sec. 6· 6·5 sec.
8 $29\frac{1}{3}$ ft. per sec.
9. 1200 ft. per sec.

Exercise VI (Page 103)

5 20 lbs.
6. $14\frac{1}{21}$ inches from the end of the tube weighing 8 oz.
7. 18°. 26′.
8. $2\frac{2}{7}$ ft. from the end nearest to 1 lb.
9. $1\frac{5}{6}$ from the heavier end.
10. 0·0773 in from the disc centre.

Exercise VII (Page 112)

2. 120 gm. 3. ·075.
4. 3·46 lbs.

Exercise VIII (Page 138)

1. 3 lbs.
2. 22°. 8′ with the horizon.
3. 12 in. from the end where the smaller force is applied.
4. 50 lbs.
5. 840 lbs
6. $373\frac{1}{3}$ lbs. 7. 21 lbs.

Exercise IX (Page 150)

1. 25.
3. 99·4 cm.
5. 0·71 sec.
6. 9·3 cm.
7. 99·39 cm. ; loses.
10. Shortened by 0·4 mm.

Exercise X (Page 168)

8. $\frac{(1760 \times 3)^2}{3 \times 12} \times 62{\cdot}5$ ft. lbs.
10. 14370384×10^4 ft. lbs.
13. $\frac{1}{2} \times 250 \times 2240 \times \frac{22}{3}$ ft. lbs.

Exercise XII (Page 196)

2. 0·0592 cm.
3. 1·13 litres.
4. 0·5 gms. per c. c.
5. $2^{\circ}.41$ C, and $5^{\circ}.4$ C.

Exercise XIII (Page 208)

2. 1 mm.

Exercise XIV (Page 227)

2. $1133{\cdot}6 \times 10^6$ dynes.
4. 680×981 dynes.
5. 625 lbs. wt.
6. 1000 gm. wt. (top).
   2000 „ „ (bottom).
   1500 „ „ (sides).

Exercise XVI (Page 249)

1. 100 cc.
2. gold—13·9 gm. ;
   silver—6·1 gm.
3. 40,008 cu. ft.
6. 30 gms. 7. 2 c. c.
10. 20 c c. ; 3·1.

Exercise XVII (Page 262)

2. 1 cm.
3. 50 gms. to be added.
5. 82 gms. 6. 360 gms.
7. 180 gms. 8. 0·8.
10. 6437·5 cu. yds. 11. 21.
12. 0·25.

Exercise XIX (Page 303)

5. 8264·46 litres.
6. $1033{\cdot}6 \times 981$ dynes.
7. 15·7 lbs. per sq. in.

Exercise XX (Page 312)

1. 1·327 gm.
2. 46·2m.
3. 0·32 c.c.
4. 1131 c.c.
5. 500 mm.
6. 500 mm.
7. $6 \times$ old pressure. 10. 2 in.

Exercise XXI (Page 333)

2. $(\frac{2}{3})^{10}$
3. 5 atmospheres. A. 40.

INDEX

THE REFERENCES ARE TO THE PAGES.

উপহার।

কোন এক

বন্ধুপত্নীর উদ্দেশে

গ্রন্থখানি

স্নেহসহকারে উৎসৃষ্ট হইল।

গ্রন্থকার।